The University of the State of New York

307TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Wednesday, August 24, 1949 — 12 m. to 3 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in trigonometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1949 or number and length in minutes of lessons taken in the summer of 1949 under a tutor licensed in the subject and supervised by the principal of the school you last attended.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1949 or an equivalent program of tutoring approved in advance by the Department is required.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 Find, to the *nearest degree,* all positive values of x less than 360° which satisfy the equation
$3 \sin^2 x - 4 \cos x + 1 = 0$ [10]

22 *a* Starting with the formulas for sin $(A-B)$ and cos $(A-B)$, derive the formula for tan $(A-B)$ in terms of tan A and tan B. [5]

b Prove the identity: $\dfrac{\cos 2x + \sin^2 x}{\cos x} = \dfrac{\cot x}{\csc x}$ [5]

23 *a* On the same set of axes, *sketch* the graphs of $y = \sin 2x$ and $y = \cos x$ as x varies from 0 to 2π radians. [5, 3]

b From the graphs sketched in answer to *a,* determine the *number* of values of x between 0 and 2π radians which satisfy the equation $\sin 2x = \cos x$ [2]

24 In $\triangle ABC$, the bisector of angle A meets BC in D. AD is represented by t, BD by m, and DC by n.

a Express t in terms of m, A and B. [2]

b Express t in terms of n, A and C. [2]

c Show that $\dfrac{m}{n} = \dfrac{\sin C}{\sin B}$ [3]

d Show that $\dfrac{m}{n} = \dfrac{AB}{AC}$ [3]

Part III

Answer at least two questions from part III.

25 In $\triangle$ *ABC*, $a = 38.2$, $b = 47.1$, $c = 56.3$; find $\angle A$ to the *nearest minute.* [10]

26 A lighthouse C is observed N 54° 10′ W of a ship which is at a certain position A. After the ship sails 500 yards due north to point B, the lighthouse is N 80° 30′ W of the ship. Find, to the *nearest yard,* the distance between ship and lighthouse when the ship is at B. [6, 4]

27 A tunnel is to be bored in a straight line from A to B, two points on opposite sides of a hill. At point C, 427 feet from A and 542 feet from B, $\angle ACB$ is found to be 71° 40′. Find, to the *nearest minute,* angle CAB. [3, 7]

28 In a certain town, two spotter stations A and B are 5660 feet apart. When an airplane passes between the stations directly above the ground line AB, an observer at A finds its angle of elevation to be 31° 30′ and at the same time an observer at B finds its angle of elevation to be 48° 10′. Find, to the *nearest 10 feet,* the altitude of the airplane at the time the observations are made. [4, 6]

Fill in the following lines:

Name of pupil...Name of school..

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed.

1 Express cos 290° as a function of a positive acute angle. 1................

2 Find the value of $\tan \frac{5\pi}{4}$ 2................

3 Find log .04566 3................

4 Find the number whose logarithm is 1.5500 4................

5 Find log tan 66° 27′ 5................

6 Find cos 31° 18′ 6................

7 In $\triangle ABC$, $a = 4$, $b = 6$, and $\cos C = \frac{1}{3}$. Find c. 7................

8 Each leg of an isosceles triangle is 18 inches and the vertex angle is 80°. Find, to the *nearest inch,* the altitude to the base. 8................

9 Find the positive value of $\cos \tan^{-1} \frac{\sqrt{13}}{6}$ 9................

10 In $\triangle ABC$, $b = 20$, $c = 12$, and $\angle A = 150°$. Find the area of $\triangle ABC$. 10................

11 In $\triangle ABC$, $c = 30$, $\sin A = .4$, and $\sin C = .6$. Find a. 11................

12 In $\triangle ABC$, $\angle C = 120°$, $a = 6$, $b = 4$. Find $\tan \frac{1}{2}(A\text{–}B)$. [Answer may be left in radical form.] 12................

13 If $\sin x = \frac{1}{\sqrt{17}}$, and x is an acute angle, find $\sin 2x$. 13................

14 If $\sin x = \frac{3}{5}$ and $\cos y = \frac{5}{13}$, and x and y are positive acute angles, find $\sin (x + y)$. 14................

15 The circumference of a circle is 12π. Find the length of an arc whose central angle is 2 radians. 15................

16 If A is a positive acute angle, express $\cot A$ in terms of $\sin A$. 16................

Directions (questions 17–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a, b* or *c*.

17 Using the data: $\angle A = 60°$, $b = 40$, $a = 36$, it is possible to construct (a) only one triangle (b) two different triangles (c) no triangle 17......

18 The statement $\sin^2 A = \frac{1 - \cos 2A}{2}$ is (a) true for all values of A (b) true for only certain values of A (c) not true for any value of A 18......

19 If x is a positive angle and $\cos x$ increases from —1 to 0, $\tan x$ (a) increases (b) decreases (c) decreases and then increases 19......

20 Sin 40° + sin 20° equals (a) cos 10° (b) $\sqrt{3}$ sin 10° (c) sin 60° 20......