June 18, 1959

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. Express cot 285° as a function of a positive acute angle. 1________

2. Express in degrees an angle of $\frac{5\pi}{12}$ radians. 2________

3. Find the number of inches in the radius of a circle in which a central angle of .4 radians subtends an arc of 1.2 inches. 3________

4. Find the positive value of cos (arc sin $\frac{2}{3}$). 4________

5. If $\tan x = \frac{1}{2}$ and $\tan y = \frac{1}{3}$, find $\tan (x + y)$. 5________

6. If A is a positive acute angle, express tan A in terms of sin A. 6________

7. Find the smallest positive value of x that satisfies the equation $\frac{\sec^2 x}{4} = 1$. 7________

8. In triangle ABC, $a = 15$, $\sin A = .3$ and $\sin B = .4$. Find b. 8________

9. In triangle ABC, $b = 8$, $c = 6$ and $\cos A = \frac{17}{32}$. Find a. 9________

10. In triangle ABC, $a = 11$, $b = 9$ and $C = 48°$. Find $\tan \frac{1}{2}(A - B)$ to the *nearest tenth.* 10________

11. In triangle ABC, $a = 10$, $b = 8$ and $C = 27°$. Find to the *nearest integer* the area of triangle ABC. 11________

12. Point A is 20 miles due north of point C. Point B is due east of C and S 39° E from A. Find to the *nearest mile* the distance from B to C. 12________

13. Find the logarithm of 0.2132. 13________

14. Find to *four* decimal places the value of cos 28° 33′. 14________

15. Find to the *nearest minute* the positive acute angle A if $\log \tan A = 0.0726$. 15________

Directions (16-20): Indicate the correct completion for *each* of the following by writing the letter *a, b, c* or *d* on the line at the right.

16. Cos $(270° + x)$ is equivalent to (a) $\sin x$ (b) $-\sin x$ (c) $\cos x$ (d) $-\cos x$ 16________

17. If x is acute, the expression $\frac{2 \sin x}{\sin 2x}$ is equivalent to (a) $\frac{2}{x}$ (b) $\frac{2}{\sin x}$ (c) $\csc x$ (d) $\sec x$ 17________

18. The maximum value of $3 \cos 2x$ is (a) $\frac{1}{3}$ (b) 2 (c) 3 (d) 6 18________

19. If x is acute, $\tan x$ equals (a) $\dfrac{1}{\cot(-x)}$ (b) $\dfrac{\sin(-x)}{\cos(-x)}$ (c) $\dfrac{\sin x}{\cos(-x)}$ (d) $\dfrac{\sin(-x)}{\cos x}$ 19________

20. The expression $\cos 3x - \cos x$ is equivalent to (a) $-\sin 2x \sin x$ (b) $-2 \sin 2x \sin x$ (c) $2 \cos 2x \cos x$ (d) $\cos 2x$ 20________

Part II

Answer three questions from this part. Show all work unless otherwise directed.

21. Find all positive values of x less than 360° that satisfy the equation $3 \cos 2x = 5 \cos x + 1$. [10]

22. *a.* Starting with a formula for $\cos 2A$, *derive* the formula for $\cos \frac{1}{2}x$ in terms of $\cos x$. [6]

b. Angle x is in quadrant IV and $\cos x = \dfrac{7}{25}$. Without the use of trigonometric tables, find $\cos \frac{1}{2}x$. [4]

23. *a.* On the same set of axes, *sketch* the graph of $y = \cos 2x$ and $y = \tan x$ as x varies from $-\dfrac{\pi}{2}$ to $\dfrac{\pi}{2}$. [4, 4]

b. From the graph made in answer to *a*, find the number of values of x between $-\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}$ for which $\tan x - \cos 2x = 0$. [2]

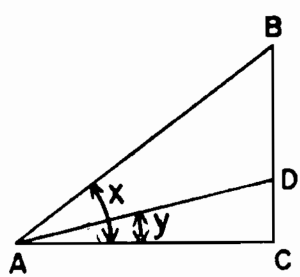
24. Prove the following identities:

a. $\dfrac{1 + \csc x}{\sec x} = \cos x + \cot x$ [4]

b. $\dfrac{\sin x}{1 - \cos x} + \dfrac{\sin x}{1 + \cos x} = 2 \csc x$ [6]

25. In the figure at the right, BC is perpendicular to AC, angle BAC is represented by x and angle DAC is represented by y.

Show that $BD = \dfrac{AB \sin(x - y)}{\cos y}$. [10]



Part III

Answer two questions from this part. Show all work.

26. In triangle ABC, $a = 230$, $b = 216$ and $c = 194$. Find angle A to the *nearest degree.* [10]

27. Point B is 47 miles N 14° E from A. Point C is S 52° E from B and N 67° E from A. Find to the *nearest mile* the distance from A to C. [6, 4]

28. In triangle ABC, angle $B = 49° 40'$, $c = 83.4$, $b = 69.5$ and angle C is obtuse. Find angle A to the *nearest ten minutes.* [10]

29. Forces of 224 pounds and 367 pounds act upon a body at an angle of 65° 20′ with each other. Find to the *nearest ten minutes* the angle which the resultant makes with the *smaller* force. [10]