Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of $\pi$ or in radical form.

1. Express cot $285^\circ$ as a function of a positive acute angle.  

2. Express in degrees an angle of $\frac{5\pi}{12}$ radians.  

3. Find the number of inches in the radius of a circle in which a central angle of $.4$ radians subtends an arc of 1.2 inches.  

4. Find the positive value of cos (arc sin $\frac{3}{5}$).  

5. If tan $x = \frac{1}{2}$ and tan $y = \frac{1}{3}$, find tan $(x + y)$.  

6. If $A$ is a positive acute angle, express tan $A$ in terms of sin $A$.  

7. Find the smallest positive value of $x$ that satisfies the equation $\frac{\sec^2 x}{4} = 1$.  

8. In triangle $ABC$, $a = 15$, sin $A = .3$ and sin $B = .4$. Find $b$.  

9. In triangle $ABC$, $b = 8$, $c = 6$ and cos $A = \frac{17}{32}$. Find $a$.  

10. In triangle $ABC$, $a = 11$, $b = 9$ and $C = 48^\circ$. Find tan $\frac{1}{2} (A - B)$ to the nearest tenth.  

11. In triangle $ABC$, $a = 10$, $b = 8$ and $C = 27^\circ$. Find to the nearest integer the area of triangle $ABC$.  

12. Point $A$ is 20 miles due north of point $C$. Point $B$ is due east of $C$ and S $39^\circ$ E from $A$. Find to the nearest mile the distance from $B$ to $C$.  

13. Find the logarithm of 0.2132.  

14. Find to four decimal places the value of cos $28^\circ 33'$.  

15. Find to the nearest minute the positive acute angle $A$ if $\log \tan A = 0.0726$.  

Directions (16-20): Indicate the correct completion for each of the following by writing the letter $a$, $b$, $c$ or $d$ on the line at the right.

16. $\cos (270^\circ + x)$ is equivalent to (a) sin $x$ (b) $-\sin x$ (c) $\cos x$ (d) $-\cos x$  

17. If $x$ is acute, the expression $\frac{2 \sin x}{\sin 2x}$ is equivalent to (a) $\frac{2}{x}$ (b) $\frac{2}{\sin x}$ (c) $\csc x$ (d) $\sec x$
18. The maximum value of $3 \cos 2x$ is (a) $\frac{1}{3}$  (b) 2  (c) 3  (d) 6  18_______

19. If $x$ is acute, $\tan x$ equals (a) $\frac{1}{\cot (-x)}$  
(b) $\frac{\sin (-x)}{\cos (-x)}$  
(c) $\frac{\sin x}{\cos (-x)}$  
(d) $\frac{\sin (-x)}{\cos x}$  19_______

20. The expression $\cos 3x - \cos x$ is equivalent to (a) $-\sin 2x \sin x$  (b) $-2 \sin 2x \sin x$  (c) $2 \cos 2x \cos x$  (d) $\cos 2x$  20_______

Part II

Answer three questions from this part. Show all work unless otherwise directed.

21. Find all positive values of $x$ less than $360^\circ$ that satisfy the equation $3 \cos 2x = 5 \cos x + 1$.  [10]

22. a. Starting with a formula for $\cos 2A$, derive the formula for $\cos \frac{1}{2}x$ in terms of $\cos x$.  [6]

b. Angle $x$ is in quadrant IV and $\cos x = \frac{7}{25}$. Without the use of trigonometric tables, find $\cos \frac{1}{2}x$.  [4]

23. a. On the same set of axes, sketch the graph of $y = \cos 2x$ and $y = \tan x$ as $x$ varies from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.  [4, 4]

b. From the graph made in answer to a, find the number of values of $x$ between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ for which $\tan x - \cos 2x = 0$.  [2]

24. Prove the following identities:

a. $\frac{1 + \csc x}{\sec x} = \cos x + \cot x$  [4]

b. $\frac{\sin x}{1 - \cos x} + \frac{\sin x}{1 + \cos x} = 2 \csc x$  [6]

25. In the figure at the right, $BC$ is perpendicular to $AC$, angle $BAC$ is represented by $x$ and angle $DAC$ is represented by $y$.

Show that $BD = \frac{\Delta B \sin (x - y)}{\cos y}$.  [10]
Part III

Answer two questions from this part. Show all work.

26. In triangle $ABC$, $a = 230$, $b = 216$ and $c = 194$. Find angle $A$ to the nearest degree. [10]

27. Point $B$ is 47 miles N $14^\circ$ E from $A$. Point $C$ is S $52^\circ$ E from $B$ and N $67^\circ$ E from $A$. Find to the nearest mile the distance from $A$ to $C$. [6, 4]

28. In triangle $ABC$, angle $B = 49^\circ 40'$, $c = 83.4$, $b = 69.5$ and angle $C$ is obtuse. Find angle $A$ to the nearest ten minutes. [10]

29. Forces of 224 pounds and 367 pounds act upon a body at an angle of $65^\circ 20'$ with each other. Find to the nearest ten minutes the angle which the resultant makes with the smaller force. [10]