June 18, 1958

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. Find the value of cos 300°. 1________

2. In a circle whose radius is 3 feet, a central angle intercepts an arc of 2 feet. Find the number of radians in the central angle. 2________

3. Express in degrees an angle of $\dfrac{2\pi}{15}$ radians. 3________

4. Find cot (arc tan 1). 4________

5. If A is a positive acute angle, express sin A in terms of tan A. 5________

6. If $\sin A = \dfrac{1}{\sec A}$, find the smallest positive value of A. 6________

7. Express $\sin 3x - \sin x$ as a product of two functions. 7________

8. If $\tan x = 2$, find $\tan 2x$. 8________

9. In triangle ABC, $a = 5$, $b = 3$ and $\sin A = \frac{1}{3}$. Find sin B. 9________

10. In triangle ABC, $a = 5$, $b = 3$ and $c = 6$. Find cos B. 10________

11. In triangle ABC, $a = 5$, $b = 3$ and $C = 100°$. Find, to the *nearest hundredth*, the value of $\tan \frac{1}{2} (A - B)$. 11________

12. A pilot in an airplane at an altitude of 3,000 feet observes the angle of depression of an airport to be 10°. How far, to the *nearest thousand feet*, is the airport from a point on the ground directly below the plane? 12________

Directions (13-20): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a, b, c* or *d*.

13. If two sides of a triangle are 10 and 20 and the angle between these sides is 65°, the area of the triangle to the *nearest integer* is (a) 42 (b) 85 (c) 91 (d) 181 13________

14. Using the data $A = 34° 20'$, $a = 55.4$ and $b = 100.0$, it is possible to construct (a) no triangle (b) a right triangle (c) two triangles (d) an obtuse triangle 14________

15. Cot $(180° - x)$ is equal to (a) tan x (b) $-$tan x (c) cot x (d) $-$cot x 15________

16. Log sin $2x$ is equal to (a) 2 log sin x (b) log 2 + log sin x (c) log 2 + log sin x + log cos x (d) log $2x$ + log sin x 16________

17. If both sin x and cos x decrease when x is increased, then x is in quadrant (a) one (b) two (c) three (d) four 17________

18. An example of an equation which is also an identity is
(a) $\cos x \csc x = 0$ (b) $\cos^2 \frac{1}{2}x - \sin^2 \frac{1}{2}x = 1$
(c) $\cos^2 \frac{1}{2}x - \sin^2 \frac{1}{2}x = \cos x$ (d) $\sin^2 x + \cos^2 x = 0$ 18______

19. The maximum value of $3 \cos 2x$ is (a) 1 (b) 2π
(c) 3 (d) 6 19______

20. If $\cos \dfrac{\pi}{3} = x - 1$, then (a) $x = \frac{1}{2}$ (b) $x = \dfrac{3}{2}$

(c) $x = \dfrac{\pi}{3} + 1$ (d) x has more than one value 20______

Part II

Answer three questions from this part. Show all work unless otherwise directed.

21. Find, to the *nearest degree,* all values of x greater than 0° but less than 360° that satisfy the equation $10 \cos 2x + 21 \sin x + 10 \sin^2 x = 0$. [10]

22. *a* Prove the following equation to be an identity: [4]

$$\frac{\cot A}{\tan A} + \frac{\tan A}{\cot A} = \frac{\cot^4 A + 1}{\cot^2 A}$$

b Show that $\dfrac{\frac{1}{2} \sin^2 x}{\sin^2 \frac{1}{2}x} - 1$ may be reduced to $\cos x$. [6]

23. *a Sketch* the graph of $y = \cos 2x$ as x varies from $-\dfrac{\pi}{2}$ to $\dfrac{\pi}{2}$ radians. [4]

b On the same set of axes used in *a, sketch* the graph of $y = 2 \sin x$ as x varies from $-\dfrac{\pi}{2}$ to $\dfrac{\pi}{2}$ radians. [4]

c From the graphs made in answer to *a* and *b*, determine the range of values of x for which the function $y = \cos 2x$ increases while the function $y = 2 \sin x$ increases also. [2]

24. List the numbers 1-5 on your answer paper. After *each* number indicate the correct completion for *each* of the following by writing the letter *a, b, c* or *d*: [10]

(1) Log 0.003472 is equal to (a) 7.5405—10 (b) 8.5405—10 (c) 7.5406—10 (d) 8.5406 -10

(2) Tan 316° 20′ is equal to (a) —1.0477 (b) —0.9545 (c) 0.9545 (d) 1.0477

(3) The smallest positive angle whose cosine is —0.8718 is (a) 119° 20′ (b) 150° 40′ (c) 209° 20′ (d) 240° 40′

(4) Log cot 25° 13′ is equal to (a) 0.3270 (b) 0.3290 (c) 9.3270—10 (d) 9.3290—10

(5) Log sin θ = 0.8557 (a) when θ = 31° 10′ (b) when θ = 58° 50′ (c) when θ = 44° 10′ (d) for no real value of θ

25. Derive the law of cosines. [Consider only the case in which the triangle is acute.] [10]

Part III

Answer two questions from this part. Show all work.

26. The sides of a triangle are 579, 914 and 1,247. Find the largest angle of the triangle to the *nearest ten minutes.* [10]

27. Two boats start at the same time from the same place. One sails due south at 12 knots and the other S 72° W at 10 knots. Find, to the *nearest degree,* the bearing of the slower boat from the faster at the end of one hour. [4, 5, 1]

28. In triangle ABC, $AB = 35$, $A = 41° 30'$ and $B = 62° 30'$. Find, to the *nearest integer,* the altitude drawn from C. [10]

29. The distance between two points A and B cannot be measured directly but is known to be about 20 yards. From a point C the distance to A is 82 yards and the distance from C to B is 64 yards. Angle CAB is 30° 40′.

a Find angle ABC to the *nearest ten minutes.* [6]

b Find, to the *nearest yard,* the distance from A to B. [4]