TRIGONOMETRY

June 18, 1958

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. Find the value of cos 300°.

2. In a circle whose radius is 3 feet, a central angle intercepts an arc of 2 feet. Find the number of radians in the central angle.

3. Express in degrees an angle of \[
\frac{2\pi}{15}
\] radians.

4. Find cot (arc tan 1).

5. If \( A \) is a positive acute angle, express sin \( A \) in terms of tan \( A \).

6. If \( \sin A = \frac{1}{\sec A} \), find the smallest positive value of \( A \).

7. Express \( \sin 3x - \sin x \) as a product of two functions.

8. If \( \tan x = 2 \), find \( \tan 2x \).

9. In triangle \( ABC \), \( a = 5 \), \( b = 3 \) and \( \sin A = \frac{1}{\sqrt{2}} \). Find \( \sin B \).

10. In triangle \( ABC \), \( a = 5 \), \( b = 3 \) and \( c = 6 \). Find \( \cos B \).

11. In triangle \( ABC \), \( a = 5 \), \( b = 3 \) and \( C = 100° \). Find, to the nearest hundredth, the value of \( \tan \frac{1}{2} (A - B) \).

12. A pilot in an airplane at an altitude of 3,000 feet observes the angle of depression of an airport to be 10°. How far, to the nearest thousand feet, is the airport from a point on the ground directly below the plane?

Directions (13-20): Indicate the correct completion for each of the following by writing on the line at the right the letter \( a \), \( b \), \( c \) or \( d \).

13. If two sides of a triangle are 10 and 20 and the angle between these sides is 65°, the area of the triangle to the nearest integer is \( (a) \, 42 \quad (b) \, 85 \quad (c) \, 91 \quad (d) \, 181 \)

14. Using the data \( A = 34° \, 20', \, a = 55.4 \) and \( b = 100.0 \), it is possible to construct \( (a) \) no triangle \( (b) \) a right triangle \( (c) \) two triangles \( (d) \) an obtuse triangle

15. \( \cot (180° - x) \) is equal to \( (a) \) \( \tan x \) \( (b) \) \(-\tan x \) \( (c) \) \( \cot x \) \( (d) \) \(-\cot x \)

16. \( \log \sin 2x \) is equal to \( (a) \) \( 2 \log \sin x \) \( (b) \) \( \log 2 + \log \sin x \) \( (c) \) \( \log 2 + \log \sin x + \log \cos x \) \( (d) \) \( \log 2x + \log \sin x \)

17. If both \( \sin x \) and \( \cos x \) decrease when \( x \) is increased, then \( x \) is in quadrant \( (a) \) one \( (b) \) two \( (c) \) three \( (d) \) four
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18. An example of an equation which is also an identity is
(a) \( \cos x \csc x = 0 \)  \hspace{1em} (b) \( \cos^2 \frac{1}{2}x - \sin^2 \frac{1}{2}x = 1 \)
(c) \( \cos^2 \frac{1}{2}x - \sin^2 \frac{1}{2}x = \cos x \)  \hspace{1em} (d) \( \sin^2 x + \cos^2 x = 0 \)  \hspace{1em} 18

19. The maximum value of \( 3 \cos 2x \) is
(a) 1  \hspace{1em} (b) \( 2\pi \)  \hspace{1em} (c) 3  \hspace{1em} (d) 6  \hspace{1em} 19

20. If \( \cos \frac{x}{3} = x - 1 \), then
(a) \( x = \frac{1}{2} \)  \hspace{1em} (b) \( x = \frac{3}{2} \)
(c) \( x = \frac{\pi}{3} + 1 \)  \hspace{1em} (d) \( x \) has more than one value  \hspace{1em} 20

Part II

Answer three questions from this part. Show all work unless otherwise directed.

21. Find, to the nearest degree, all values of \( x \) greater than \( 0^\circ \) but less than \( 360^\circ \) that satisfy the equation \( 10 \cos 2x + 21 \sin x + 10 \sin^2 x = 0 \).  \hspace{1em} [10]

22. a Prove the following equation to be an identity:
\[
\frac{\cot A}{\tan A} + \frac{\tan A}{\cot A} = \frac{\cot^4 A + 1}{\cot^2 A}
\]
\[
\frac{1}{2} \sin^2 x = \frac{\cot A}{\tan A} + \frac{\tan A}{\cot A}
\]
\[b \text{ Show that } \frac{1}{2} \sin^2 x - 1 \text{ may be reduced to } \cos x. \]  \hspace{1em} [6]

23. a Sketch the graph of \( y = \cos 2x \) as \( x \) varies from \( -\frac{\pi}{2} \) to \( \frac{\pi}{2} \) radians.  \hspace{1em} [4]

b On the same set of axes used in a, sketch the graph of \( y = 2 \sin x \) as \( x \) varies from \( -\frac{\pi}{2} \) to \( \frac{\pi}{2} \) radians.  \hspace{1em} [4]

c From the graphs made in answer to a and b, determine the range of values of \( x \) for which the function \( y = \cos 2x \) increases while the function \( y = 2 \sin x \) increases also.  \hspace{1em} [2]

24. List the numbers 1-5 on your answer paper. After each number indicate the correct completion for each of the following by writing the letter a, b, c or d:  \hspace{1em} [10]

1. \( \log 0.003472 \) is equal to
(a) 7.5405-10  \hspace{1em} (b) \( 8.5406 \) \hspace{1em} (c) 7.5406-10  \hspace{1em} (d) \( 3.5405 \) \hspace{1em} (e) 8.5405-10

2. \( \tan 316^\circ 20' \) is equal to
(a) \(-1.0477\)  \hspace{1em} (b) \(-0.9545\)  \hspace{1em} (c) 0.9545  \hspace{1em} (d) \(1.0477\)

3. The smallest positive angle whose cosine is \(-0.8718\) is
(a) \(119^\circ 20'\)  \hspace{1em} (b) \(150^\circ 40'\)  \hspace{1em} (c) \(209^\circ 20'\)  \hspace{1em} (d) \(240^\circ 40'\)

4. \( \log \cot 25^\circ 13' \) is equal to
(a) \(0.3270\)  \hspace{1em} (b) \(0.3290\)  \hspace{1em} (c) \(9.3270\)  \hspace{1em} (d) \(9.3290-10\)

5. \( \log \sin \theta = 0.8557 \) when \( \theta = 31^\circ 10' \) when \( \theta = 58^\circ 50' \) when \( \theta = 44^\circ 10' \) for no real value of \( \theta \)
25. Derive the law of cosines. [Consider only the case in which the triangle is acute. ] [10]

Part III

Answer two questions from this part. Show all work.

26. The sides of a triangle are 579, 914 and 1,247. Find the largest angle of the triangle to the nearest ten minutes. [10]

27. Two boats start at the same time from the same place. One sails due south at 12 knots and the other S 72° W at 10 knots. Find, to the nearest degree, the bearing of the slower boat from the faster at the end of one hour. [4, 5, 1]

28. In triangle ABC, AB = 35, \( A = 41° 30' \) and \( B = 62° 30' \). Find, to the nearest integer, the altitude drawn from C. [10]

29. The distance between two points \( A \) and \( B \) cannot be measured directly but is known to be about 20 yards. From a point \( C \) the distance to \( A \) is 82 yards and the distance from \( C \) to \( B \) is 64 yards. Angle \( CAB \) is 30° 40'.
   \( a \) Find angle \( ABC \) to the nearest ten minutes. [6]
   \( b \) Find, to the nearest yard, the distance from \( A \) to \( B \). [4]