Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. Express sec 290° as a function of a positive acute angle.

2. Express in radians an angle of 100°.

3. Find the smallest positive value of \( A \) which satisfies the equation \( \tan^2 A - 3 = 0 \).

4. In triangle \( ABC \), angle \( C = 90° \), side \( a = 18 \) and side \( b = 12 \). Find \( \tan \frac{1}{2} (A - B) \).

5. Find the positive value of \( \cos \left\{ \sin^{-1} \frac{5}{13} \right\} \).

6. If \( \tan \theta = \frac{1}{2} \), find \( \sin 2\theta \).

7. Express \( \sin^2 \theta \) in terms of \( \cos \theta \).

8. Find the antilogarithm of 1.8362.

9. Find \( \log \cot 53° 27' \).

10. The legs of a right triangle are 12 and 24. Find to the nearest minute the smaller acute angle of the triangle.

Directions (11-16): Indicate the correct completion for each of the following by writing the letter \( a \), \( b \) or \( c \) on the line at the right.

11. The expression \( \frac{\tan \theta + \sec \theta}{1 + \sin \theta} \) can be reduced to

   \( \frac{1}{\cos \theta} \)

(a) \( \cos \theta \)   (b) \( \cot \theta \)   (c) \( \sin \theta \)

12. The expression \( \frac{\sin (45° + x)}{\sqrt{2} + \sin x} \) is always equal to

   \( \frac{\sin x - \cos x}{\sqrt{2}} \)

(a) \( \frac{\cos x + \sin x}{\sqrt{2}} \)   (b) \( \frac{\cos x - \sin x}{\sqrt{2}} \)   (c) \( \frac{\sin x}{\sqrt{2}} \)

13. Using the data \( A = 43° \), \( a = 15 \) and \( b = 24 \), it is possible to construct (a) no triangle (b) only one triangle (c) two triangles

14. The expression \( \sin^2 x - \cos^2 x \) is always equal to

(a) \( \cos (-2x) \)   (b) \(-1\)   (c) \(-\cos 2x\)

15. If, in triangle \( ABC \), \( a = 3 \), \( b = 5 \) and \( c = 6 \), the cosine of the largest angle is equal to

(a) \(-1/15\)   (b) \(-5/6\)   (c) \(1/9\)

16. The graph of the function \( y = 2 \sin \frac{\pi}{2} x \) passes through the point whose coordinates are

(a) \( (\pi, 2) \)   (b) \( \left(\frac{\pi}{2}, 1\right) \)

(c) \( (\pi, 1) \)
Directions (17-20): For each of the following, tell whether the statement is always true, sometimes true or never true by writing the word always, sometimes or never on the line at the right.

17. \( \cos (270^\circ - \theta) = -\sin \theta \).  
18. \( \sin^2 2x + \cos^2 2x = 2 \).
19. \( \cos 4A - \cos 2A = -2 \sin 3A \sin A \).
20. \( \cot^2 A - \cot A = 0 \).

Part II

Answer three questions from this part. Show all work unless otherwise directed.

21. a Prove the identity: \( \frac{\cot^2 \theta}{\csc \theta - 1} = \frac{1 + \sin \theta}{\sin \theta} \). [6]

b Find the smallest positive value of \( A \) which satisfies the equation
\[ 2 \sin^2 A + \sin A - 1 = 0. \] [4]

22. Answer both a and b without the use of trigonometric tables.

a Angles \( x \) and \( y \) are acute, \( \sin x = 4/5 \) and \( \cos y = 5/13 \). Find the value of \( \tan (x + y) \). [5]

b If \( A = \tan^{-1} \frac{2}{\sqrt{5}} \), find the value of \( \sin 2A \).

[Answer may be left in radical form.] [5]

23. Derive the law of cosines. [Consider only the case in which the triangle is acute.] [10]

24. a On the same set of axes sketch the graph of \( y = \sin x \) and \( y = \cos x \) as \( x \) varies from 0 to \( 2\pi \) radians. [3, 3]

b From the graphs made in answer to part a, determine the smallest value of \( x \) greater than 0 and less than \( 2\pi \) radians for which

(1) \( \sin x + \cos x = 1 \) [2]

(2) \( \sin x + \cos x = -1 \) [2]

25. \( P \) is a point whose rectangular coordinates are represented by \( x \) and \( y \) as shown in the drawing. The distance from the origin \( O \) to point \( P \) is represented by \( r \), and the angle that \( OP \) makes with the positive portion of the \( x \)-axis is represented by \( \theta \).

a Express \( x \) in terms of \( r \) and \( \theta \). [1]

b Express \( y \) in terms of \( r \) and \( \theta \). [1]

c Show that the equation
\[ x^2 + y^2 - 2x = 0 \] can be reduced to the form \( r = 2 \cos \theta \). [8]
Answer two questions from this part. Show all work.

26. In triangle $ABC$, angle $C = 78^\circ$, side $AC = 150$ feet and side $BC = 200$ feet. Find angle $A$ to the nearest degree. [10]

27. In triangle $ABC$, angle $C = 90^\circ$, angle $CAB = 48^\circ$ and side $AB = 285$. $P$ is a point on side $CB$ and angle $CAP = 22^\circ$. Find $PB$ to the nearest integer. [10]

28. A plot of ground has the form of the quadrilateral $ABCD$ shown at the right. Side $AB = 36$ rods, side $AD = 48$ rods, angle $DAB = 90^\circ$, angle $DBC = 50^\circ$ and side $BC = 28$ rods. Find to the nearest square rod the area of the plot. [10]

29. Point $B$ is 12 miles directly east of point $A$. Point $C$, which is north and east of $A$, is 18 miles from $A$ and 14 miles from $B$. Find to the nearest degree the bearing of $C$ from $A$. [10]