

The University of the State of New York

288TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, June 17, 1943—9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 Starting with the formulas for $\sin(x + y)$ and $\cos(x + y)$, derive the formula for $\tan 2x$ in terms of $\tan x$. [10]

22 a Sketch the graph of $y = \cos x$ as x varies from 0° to 360° . [4]

b By means of the graph made in answer to a, show that there are always four values of x between 0° and 360° which satisfy the equation $\cos^2 x = k$ where k is a positive number less than 1. [6]

23 From two points due west of a captive balloon, the angles of elevation of the balloon are x and y ($x > y$). The distance between the two points is d . Show that the distance s from the point on the ground directly beneath the balloon to the nearer point of observation is given by

the formula $s = \frac{d \cos x \sin y}{\sin(x - y)}$ [10]

24 Prove that the area of any quadrilateral is equal to one half the product of the two diagonals multiplied by the sine of the included angle. [10]

Part III

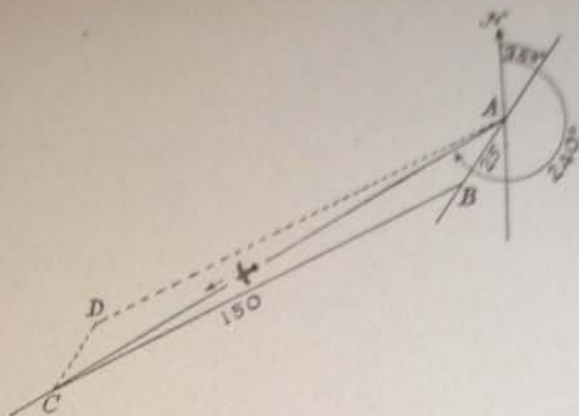
Answer at least one question from part III.

25 From the top of a house 32 feet high, the angle of elevation of the top of a pole is $12^\circ 10'$; at the ground level of the house, the angle is $40^\circ 50'$. Find, correct to the nearest foot, the height of the pole. [10]

26 A merchant vessel sails from a certain port directly east at 12 knots. A submarine is 10 nautical miles S. W. from this port. At what rate must the submarine proceed in order to overtake the vessel in 2 hours? [Express answer to the nearest knot.] [1 knot = 1 nautical mile per hour] [10]

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27 A pilot whose plane has an air speed of 150 m.p.h. finds that he is to fly a course of 240° . The wind speed is 25 m.p.h. and is blowing from 35° . These data are represented in the drawing at the right, where $ABCD$ is a parallelogram. Find the heading (reflex angle NAD) that the pilot should take. [Express the result to the nearest degree.] [10]



Part IV

Answer at least one question from part IV.

28 Given right spherical triangle ABC in which C is the right angle, with angle $A = 78^\circ$ and side $c = 115^\circ$. Find B correct to the nearest minute. [10]

29 Given spherical triangle ABC whose sides a , b and c are 55° , 70° and 105° respectively. Find angle C correct to the nearest minute. [10]

[Suggestion: The formula $\tan \frac{C}{2} = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s \sin(s-c)}}$, in which s represents the semiperimeter, may be used.]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

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|--|---------|
| 1 Find the logarithm of 0.5473 | 1..... |
| 2 Find, correct to the nearest hundredth, the number whose logarithm is 1.6733 | 2..... |
| 3 Find $\log \cos 38^\circ 22'$ | 3..... |
| 4 Find, correct to the nearest minute, the angle whose tangent is 1.4400 | 4..... |
| 5 Express $\sin 190^\circ$ as a function of a positive angle less than 45° | 5..... |
| 6 Given that A is a positive acute angle and that $\cos A = \frac{1}{4}$; express in radical form the value of $\tan \frac{1}{2} A$ | 6..... |
| 7 In triangle ABC , $b = 3$, $c = 4$ and $A = 60^\circ$. Find a . [Answer may be left in radical form.] | 7..... |
| 8 In triangle ABC , $b = 5$ and $c = 3$. Find the value of the ratio $\tan \frac{1}{2} (B + C) : \tan \frac{1}{2} (B - C)$ | 8..... |
| 9 In triangle ABC , $A = 45^\circ$ and $B = 30^\circ$. Find the value of the ratio $a : b$. [Answer may be left in radical form.] | 9..... |
| 10 An interior angle of a regular polygon contains 135° and the apothem of the polygon is 5. Find, correct to the nearest integer, a side of the polygon. | 10..... |
| 11 Express in radian measure the positive acute angle which satisfies the equation $\sin x = \cos x$ | 11..... |
| 12 Express in mils an angle of 90° . | 12..... |
| 13 Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. | 13..... |
| 14 Express $\tan (x - y)$ in terms of $\tan x$ and $\tan y$. | 14..... |
| 15 Is there a right spherical triangle in which $b = 40^\circ$ and $B = 100^\circ$? [Answer yes or no.] | 15..... |
| 16 State whether the following statement is true or false: The polar triangle of a quadrantal triangle is a right spherical triangle. | 16..... |
| 17 In right spherical triangle ABC in which C is the right angle, sides a and b are known. Write the formula that should be used to find side c . | 17..... |
| Directions (questions 18-20) — Indicate the correct answer to each question by writing the letter a , b or c on the line at the right. | |
| 18 As angle A increases from 0 radians to π radians, (a) $\cos A$ increases from 0 to +1 and then decreases from +1 to 0, (b) $\cos A$ decreases from +1 to -1, (c) $\cos A$ decreases from +1 to -1 and then increases from -1 to +1. | 18..... |
| 19 The statement $\cos 3x = \cos^2 \frac{3x}{2} - \sin^2 \frac{3x}{2}$ is (a) true for all values of x , (b) not true for any value of x , (c) true for only certain values of x . | 19..... |
| 20 Using the values $A = 40^\circ$, $a = 10$ and $b = 8$, there can be constructed (a) no triangle, (b) only one triangle, (c) two triangles. | 20..... |