TRIGONOMETRY

Tuesday, June 13, 1911 — 1:15 to 4:15 p.m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.
The minimum time requirement in either plane trigonometry or spheric trigonometry is one recitation a week for a school year or two recitations a week for half a school year.

To receive credit for plane trigonometry students should answer three questions from group I and three questions from group II.
To receive credit for spheric trigonometry students should answer three questions from group I and three questions from group III.
Students who pass spheric trigonometry will receive credit for plane trigonometry also.

Group I
1. Express cos x, sec x and tan x in terms of tan \( \frac{1}{2} x \).
2. Find the values of \( A \) between 0° and 360° that satisfy the equation \( \sin A + \cos A = \sqrt{2} \).
3. Prove that \( \sin 45° = \frac{1}{\sqrt{2}} \); tan 30° = \( \frac{1}{\sqrt{3}} \); cos 60° = \( \frac{1}{2} \).
4. Solve, for the value of \( x \), the equation \( \sin^2 x - \cos^2 x = \frac{1}{2} \). Verify the result.

Group II
5. Prove that in a plane triangle \( a = b \cos C + c \cos B \).
6. From two points \( A \) and \( C \), 20 feet apart, a buoy \( B \) is observed; the angle \( CAB \) is 104°, the angle \( ACB \) is 57°. How far is the buoy from \( A \)?
7. Given \( b = 420 \), \( a = 540 \), \( C = 52° 6' \); find the angle \( A \).
8. Complete and prove \( \sin (a + b) = \)

Group III
9. Deduce the following formulas for right spheric triangles [simply applying Napier's rule is not sufficient]:
   \[
   \cos A = \cos a \sin B \\
   \cos A = \tan b \cot c
   \]
10. Prove that if in a right spheric triangle the two oblique angles are both greater than 90°, the hypotenuse is less than 90°.
11. In a right spheric triangle given \( c = 70° 30' \), \( A = 100° \); find \( a \) and \( b \).
12. Given \( a = 58° \), \( b = 137° 20' \), \( B = 131° 20' \); find \( A \) and \( C \).