Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of $\pi$ or in radical form.

1. Find the numerical value of $\tan \frac{\pi}{3}$.

2. If $B$ is a positive acute angle and $\cos 30^\circ = \frac{1}{\sec B}$, find the number of degrees in angle $B$.

3. Express $4.5\pi$ radians in degrees.

4. If $\sin A = \frac{1}{2}$ and $\sec A$ is negative, evaluate $\tan A$.

5. Find $\log \tan 27^\circ 34'$.

6. Find to the nearest minute the smallest positive angle whose cosine is 0.9525.

7. If $x$ is a positive acute angle, express $\tan x$ in terms of $\csc x$.

8. Evaluate $\cos (270^\circ - A)$, if $\sin A = \frac{1}{3}$ and angle $A$ is acute.

9. In a circle a central angle intercepts an arc equal in length to the diameter of the circle. How many radians are there in this central angle?

10. If $2 \sin 3x \cos 3x = 1$ and $x$ is a positive angle, find in degrees the smallest value of $x$.

11. Express $\tan \frac{\theta}{2}$ in terms of $\cos \theta$.

12. Express in degrees the value of $x$ between $180^\circ$ and $360^\circ$ which satisfies the equation $\sin x = \csc x$.

13. If $\tan \left(\frac{\pi}{2} + y\right) = 4$ and $\tan y = 2$, find the numerical value of $\tan x$.

14. If $4x$ is a positive acute angle and $\sin 4x = \cos x$, find the number of degrees in $x$.

15. If $y$ is a positive angle less than $180^\circ$, find in degrees the value of $y$ for which $y = \arctan 1$. 

16. 

17. 

18. 

19. 

20.
16. Points A (4, 2) and B (8, 5) lie on line l as shown in the accompanying figure. Find the tangent of the acute angle \( \theta \) which line \( l \) makes with the positive x-axis.

17. The sides of a triangle are 6, 10 and 14. Compute the value of the cosine of the smallest angle of the triangle.

18. Find the area of triangle \( ABC \) in which \( AB = 20, BC = 10 \) and angle \( B = 45^\circ \).

19. Express \( \cos 295^\circ \) as a function of a positive acute angle.

20. In triangle \( ABC \), \( a = 10, b = 8 \) and \( \sin B = \frac{3}{4} \). Find \( \sin A \).

21. If \( x \) and \( y \) are acute angles, and if \( \sin x = \frac{12}{13} \) and \( \sin y = \frac{3}{5} \), find the value of \( \sin (x + y) \).

22. Solve for \( x \): \( \log (x - 1) + \log (x + 3) = \log (x^2) \)

23. What is the period of the graph which represents the function \( y = 3 \cos \frac{1}{2}x \)?

24. The base of an isosceles triangle is 10 and the altitude to the base is 12. Find the sine of a base angle.

25. In triangle \( ABC \), \( a = 6, c = 4 \) and angle \( B = 76^\circ \). Find to the nearest hundredth the numerical value of \( \tan \left( \frac{A - C}{2} \right) \).

Directions (26-30): Indicate the correct completion for each of the following by writing on the line at the right the number 1, 2, 3 or 4.

26. The expression \( \sin x - \sin y \) is equivalent to (1) \( \sin (x - y) \)
(2) \( \frac{2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}}{2} \)
(3) \( \frac{2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}}{2} \)
(4) \( \sin x \cos y - \cos x \sin y \)

27. The statement \( \sin x + \cos x = 1 \) is true for (1) only one value of \( x \)
(2) more than one value of \( x \), but not all values
(3) all values of \( x \)
(4) no values of \( x \)

28. If the data \( A = 60^\circ, a = 15 \) and \( b = 20 \) are used, (1) triangle \( ABC \) must be acute
(2) triangle \( ABC \) must be obtuse
(3) triangle \( ABC \) may be either acute or obtuse
(4) no triangle can be constructed
29. The expression \( \sin 3\theta \cos \theta - \cos 3\theta \sin \theta \) is equivalent to
(1) \( \cos \theta \)  (2) \( \sin \theta \)  (3) \( \sin 2\theta \)  (4) \( \cos^2 \theta - \sin^2 \theta \)  

30. As angle \( A \) increases from \( 45^\circ \) to \( 225^\circ \), \( \sin A \) (1) increases throughout the interval (2) increases, then decreases (3) decreases throughout the interval (4) decreases, then increases

**Part II**

*Answer four questions from this part. Show all work unless otherwise directed.*

31. \( a \) Find all nonnegative values of \( \theta \) less than \( 360^\circ \) which satisfy the equation \( \sin 2\theta + \cos (90^\circ - \theta) = 0 \). [8]

   \( b \) What value of \( \theta \) is greater than \( 360^\circ \) and less than \( 540^\circ \) satisfies the equation in part \( a \)? [2]

32. \( a \) Starting with the formula for \( \cos (x + y) \), derive a formula for \( \cos 2x \) in terms of \( \cos x \). [4]

   \( b \) In the accompanying figure, \( ABC \) is a triangle and \( AD \) is a line meeting \( BC \) in \( D \). If \( BD = r \), \( DC = s \), \( AB = c \), \( AC = b \), angle \( BAD = x \) and angle \( DAC = y \), show that \( \frac{\sin x}{\sin y} = \frac{br}{cs} \). [6]

33. \( a \) Sketch and label the graph of \( y = \frac{1}{2} \sin 2x \) as \( x \) varies from \( -\frac{\pi}{2} \) radians to \( +\frac{3\pi}{2} \) radians, inclusive. [6]

   \( b \) On the same set of axes, sketch and label the graph of \( y = \cos x \) as \( x \) varies from \( -\frac{\pi}{2} \) radians to \( +\frac{3\pi}{2} \) radians, inclusive. [3]

   \( c \) How many values of \( x \) between 0 and \( \pi \) radians satisfy the equation \( \frac{1}{2} \sin 2x = \cos x ? \) [1]

34. \( a \) Prove that the following equality is true for all values of \( x \) for which the members are defined: (In other words, prove the equality is an identity.) [8]

\[
\frac{1}{\cos 2x} = \frac{2}{1 - \tan^2 x} = 1
\]

\( b \) Division by zero is undefined. Determine two positive values of \( x \) less than \( 180^\circ \) for which \( \frac{1}{\cos 2x} \), the left member of the equality in part \( a \), is undefined. [2]
35. \( a \) In triangle \( ABC \), angle \( B \), angle \( C \) and side \( a \) are given. Derive a formula for \( h \), the altitude drawn to side \( a \), in terms of the given parts \( B \), \( C \) and \( a \). [6]

\( b \) Find to the nearest tenth the altitude to side \( a \) in triangle \( ABC \) if angle \( B = 30^\circ \), angle \( C = 46^\circ \) and \( a = 10 \). [4]

36. Answer either \( a \) or \( b \):

\( a \) Two forces of 720 pounds and 640 pounds, respectively, act upon a body at an angle of 42° 40' with each other. Find to the nearest ten minutes the angle that the resultant force makes with the smaller force. [10]

\( OR \)

\( b \) Acute triangle \( ABC \) is inscribed in a circle. Angle \( B \) intercepts an arc of 142°, \( AB = 17.5 \) and \( AC = 23.8 \).

(1) Find to the nearest degree the three angles of triangle \( ABC \). [8]

(2) Name the longest side of triangle \( ABC \). [2]