The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Wednesday, January 25, 1961 — 1:15 to 4:15 p.m., only

Name of pupil...Name of school...

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Find the numerical value of $\csc(-30°)$. 1...............

2 Find the numerical value of $\cos \dfrac{4\pi}{3}$. 2...............

3 If $\tan \theta = m$, express $\tan(180° - \theta)$ in terms of m. 3...............

4 If x is an acute angle, express $\sec x$ in terms of $\sin x$. 4...............

5 Find the number whose logarithm is 1.8752. 5...............

6 Find log cot 50° 24′. 6...............

7 Find to the *nearest minute* the positive acute angle A if $\sin A = 0.6122$. 7...............

8 The sides of a triangle are 6, 8 and 10. Find the value of the cosine of the largest angle. 8...............

9 Find the value of $\tan(\text{arc} \cot \frac{1}{2})$. 9...............

10 Find the number of degrees in angle x if $\sin x = -\frac{1}{2}$ and $\cos x = \dfrac{\sqrt{3}}{2}$. 10...............

11 Find the number of inches in the length of an arc of a circle intercepted by a central angle of $\frac{1}{2}$ radian, if the diameter of the circle is 6 inches. 11...............

12 If $\cos x = \frac{1}{2}$, find the value of $\cos 2x$. 12...............

13 Find in degrees the smallest positive value of x that satisfies the equation $\sin \frac{1}{3}x = \frac{1}{2}$. 13...............

14 The leg of an isosceles triangle is 6 inches in length and the vertex angle is 120°. Find the number of square inches in the area of the triangle. 14...............

15 A is 50 miles due north of B; C is 40 miles due east of A. Find to the *nearest degree* the direction of C from B. 15...............

Directions (16–27): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

16 In triangle ABC, the number of degrees in A is twice the number of degrees in B. Then $\frac{a}{b}$ equals

(1) $2 \sin B \cos B$ (2) 2 (3) $2 \sin B$ (4) $2 \cos B$ 16......

17 Two sides of a triangle are a and b and the included angle $C = 60°$. Side c is equal to

(1) $\sqrt{a^2+b^2-ab}$
(2) $\sqrt{a^2+b^2-2ab}$
(3) $\sqrt{a^2+b^2+ab}$
(4) $\sqrt{a^2+b^2+2ab}$ 17......

18 If A is an acute angle, log cot A equals

(1) $\log \cos A - \log \sin A$
(2) $\frac{\log \cos A}{\log \sin A}$
(3) $\log \sin A - \log \cos A$
(4) $\frac{\log \sin A}{\log \cos A}$ 18......

19 $\sin 3x + \sin x$ equals

(1) $2 \sin 2x \sin x$
(2) $2 \cos 2x \cos x$
(3) $2 \sin 2x \cos x$
(4) $2 \cos 2x \sin x$ 19......

20 The maximum value of $\frac{1}{4} \cos 2x$ is (1) 1 (2) 2 (3) $\frac{1}{2}$ (4) $\frac{1}{4}$ 20......

21 A student is given the following information: $A = 30°$, $a = 10$ and $b = 12$. He tries to determine the number of degrees in B that, with the given parts, will form a triangle ABC. He should discover that there

(1) is no value of B which satisfies the given conditions
(2) is exactly one value of B which satisfies the given conditions
(3) are two distinct values of B, each less than 90°, which satisfy the given conditions
(4) are two values of B, one less than 90°, the other greater than 90°, which satisfy the given conditions 21......

22 In triangle ABC, $A = 60°$, $b = 10$ and $c = 6$. Tan $\frac{1}{2}(B - C)$ equals

(1) $\frac{\sqrt{2}}{4}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{\sqrt{2}}{12}$ (4) $\frac{\sqrt{3}}{12}$ 22......

23 $\tan (45° - \theta)$ equals

(1) $\frac{1 - \tan \theta}{1 + \tan \theta}$
(2) $\frac{\tan \theta - 1}{\tan \theta + 1}$
(3) $\cot \theta$
(4) $\frac{1}{\cot \theta}$ 23......

24 Which value(s) of x between 0° and 360° would satisfy the equation $\sin x = \cos x$?

(1) 45° only
(2) 45° and 135°
(3) 45° and 225°
(4) 45° and 315° 24......

25 As x increases from 0 to 2π radians, $\sin x$ increases
(1) in quadrant I only
(2) in quadrants I and II
(3) in quadrants I and III
(4) in quadrants I and IV
25......

26 $\sin x$ is always equal to
(1) $\sin\left(\frac{\pi}{2} + x\right)$
(2) $\sin(2\pi + x)$
(3) $\sin\left(\frac{\pi}{2} - x\right)$
(4) $\sin(2\pi - x)$
26......

27 $\cos(A - B)$ is equal to
(1) $\cos A \cos B - \sin A \sin B$
(2) $\cos A \cos B + \sin A \sin B$
(3) $\sin A \cos B - \cos A \sin B$
(4) $\sin A \cos B + \cos A \sin B$
27......

Directions (28–30): Indicate whether the following statements are true for
(1) all real values of x,
(2) some, but not all, real values of x,
(3) no real value of x,
by writing on the line at the right the *number* 1, 2 or 3.

28 $\sin(270° - x) = -\cos x$ 28......

29 $2\sin^2 x + 2\cos^2 x = 1$ 29......

30 $\cos x = 2\cos^2 \frac{1}{2}x - 1$ 30......

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Find the value(s) of $\tan x$ that would satisfy the equation $2\sec^2 x + 5\tan x = 0$. [6]
b Express the value(s) of x in inverse trigonometric form. [2]
c How many values of x between 0° and 360° satisfy the given equation? [2]

32 *a* Starting with formulas for $\sin(A - B)$ and $\cos(A - B)$, *derive* the formula
$\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$. [4]
b By substituting the values $A = 45°$ and $B = 30°$ in the formula for $\tan(A - B)$, *show* that $\tan 15° = 2 - \sqrt{3}$. [6]

33 *a* *Sketch* the graph of $y = \frac{1}{2}\sin 2x$, as x varies from $-\pi$ radians to $+\pi$ radians. [8]
b What is the period of the graph sketched in part *a*? [2]

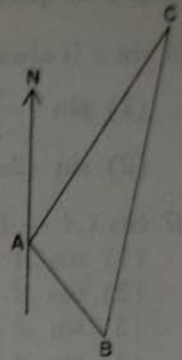
34 *a* Starting with the formula for the area of a triangle $K = \frac{1}{2}ab\sin C$,
derive the formula $K = \dfrac{a^2 \sin B \sin C}{2\sin A}$. [5]
b Prove that the following equality is an identity: [5]

$$\tan\theta + \cot\theta = \frac{2}{\sin 2\theta}$$

[OVER]

35 In triangle ABC, $A = 48°\ 20'$, $a = 30$ feet, $b = 36$ feet and B is an obtuse angle. Find C to the *nearest ten minutes*. [10]

36 Two airports, A and B, are 600 miles apart. The bearing of B from A is 140° (S 40° E). Two planes leave points A and B simultaneously at speeds of 240 m.p.h. and 300 m.p.h., respectively, and they both reach point C in 5 hours. Find to the *nearest degree* the course (angle NAC) followed by the plane that took off from A. [Assume that there is no wind.] [4,6]



FOR TEACHERS ONLY

T

INSTRUCTIONS FOR RATING
TRIGONOMETRY

Wednesday, January 25, 1961 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 16–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) -2

(2) $-\frac{1}{2}$

(3) $-m$

(4) $\dfrac{1}{\sqrt{1-\sin^2 x}}$

(5) 75.02

(6) 9.9177—10

(7) 37° 45′

(8) 0

(9) 2

(10) 330

(11) $1\frac{1}{2}$

(12) $-\frac{1}{2}$

(13) 90

(14) $9\sqrt{3}$ *or* 15.6

(15) 39° *or* N 39° E

(16) 4

(17) 1

(18) 1

(19) 3

(20) 4

(21) 4

(22) 2

(23) 1

(24) 3

(25) 4

(26) 2

(27) 2

(28) 1

(29) 3

(30) 1

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10%, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(31) *a* $\tan x = -\frac{1}{2}$, $\tan x = -2$ [6]

b $x = \text{arc tan } (-\frac{1}{2})$
$x = \text{arc tan } (-2)$ [2]

c 4 [2]

(33) *b* 180° *or* π radians [2]

(35) $\angle C = 15° \; 20'$ [10]

(36) Analysis [4]

32° *or* N 32° E [6]