Part I

Answer all questions in this part. Each correct answer will receive \(2\frac{1}{2}\) credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of \(\pi\) or in radical form.

1. Express \(\sec 140^\circ\) as a function of a positive acute angle.

2. Find the value of \(\cot \frac{5\pi}{4}\).

3. Find the number of degrees in the smallest positive angle \(x\) if \(\cos x = \frac{\sqrt{2}}{2}\) and \(\tan x\) is negative.

4. Express in terms of \(a\) the positive value of \(\cos (\text{arc} \sin a)\).

5. Find \(\sin 57^\circ 24'\).

6. Find, to the nearest minute, the positive acute angle \(A\), if \(\log \tan A = 9.7820-10\).

7. In triangle \(ABC\), \(a = 5\), \(A = 30^\circ\), \(C = 45^\circ\). Find side \(c\).

8. In triangle \(ABC\), \(a = 5\), \(b = 6\). Express \(\cos C\) in terms of side \(c\).

9. Express \(\sin 70^\circ - \sin 40^\circ\) as a product of two functions.

10. If \(x\) is in the first quadrant and \(\cos x = \frac{7}{25}\), find the value of \(\cos \frac{1}{2}x\).

11. Two forces of 5 pounds and 12 pounds act upon a body at an angle of 90° with each other. Find in pounds the magnitude of the resultant.

12. How many triangles can be constructed using the data \(A = 30^\circ\), \(b = 12\) and \(a = 5\)?

[over]
Directions (13–16) : Indicate the correct completion for each of the following by writing on the line at the right the letter a, b, c or d.

13 If, in triangle ABC, \( a = 2b \), then \( \frac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)} \) is equal to
   (a) 6   (b) 2   (c) 3   (d) 4  13.

14 The maximum value of \( 2 \sin x \cos x \) is  \( \frac{\log \sin x}{\log \cos x} \)
   (a) 1   (b) 2   (c) \( \frac{1}{2} \)   (d) 4  14.

15 If \( x \) is an acute angle, \( \log \tan x \) is equal to
   (a) \( \frac{\log \sin x}{\log \cos x} \)
   (c) \( \log \sin x - \log \cos x \)
   (b) \( \frac{\log \cos x}{\log \sin x} \)
   (d) \( \log \cos x - \log \sin x \)  15.

16 In a circle whose radius is \( r \) inches, a central angle of two radians intercepts an arc of
   (a) 2 inches   (b) \( 2r \) inches   (c) \( 2r^2 \) inches   (d) \( 2\pi r \) inches  16.

Directions (17–20) : Indicate whether the following statements are true for (a) all real values of \( x \), (b) some but not all real values of \( x \) or (c) no real value of \( x \), by writing on the line at the right the letter a, b or c.

17 \( \sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} = \frac{1}{2} \).  17.

18 \( \sec x = \frac{1}{2} \sin x \).  18.

19 \( \cos 2x = \cos^2 x - \sin^2 x \).  19.

20 \( \sin x - \cos x = 0 \).  20.
Part II

Answer three questions from this part. Show all work unless otherwise directed.

21 Find all positive values of \( x \) less than 360° which satisfy the equation 
\[
\cos 2x - 3 \cos x - 1 = 0. \quad [10]
\]

22 a On the same set of axes, sketch the graphs of \( y = \cos 2x \) and \( y = 2 \cos x \), as \( x \) varies from 0 to 2\( \pi \) radians. \([4, 4]\)

b From the graphs made in answer to part a, determine the maximum value of 
\[
\cos 2x - 2 \cos x. \quad [2]
\]

23 a Prove the identity: 
\[
\tan x + \cot x = \sec x \csc x \quad [5]
\]

b In triangle \( ABC \), \( B = 45° \). Using the fact that \( C = 180° - (A + B) \), show that 
\[
\sin C = \frac{\sqrt{2}}{2} (\sin A + \cos A). \quad [5]
\]

24 a Starting with the formula for \( \tan (x + y) \), derive 
(1) the formula for \( \tan (x - y) \) by substituting \((-y)\) for \( y \) \([2]\)
(2) the formula for \( \tan 2x \) \([2]\)

b Using the formula for \( \tan (x - y) \), in which \( x = 45° \) and \( y = 30° \), express \( \tan 15° \) in radical form. \([3]\)

c Using the formula for \( \tan 2x \), and given that \( \tan 30° = \frac{\sqrt{3}}{3} \), show that \( \tan 60° = \sqrt{3} \). \([3]\)

25 A surveyor wishes to determine the height \( CD \) of a vertical building. \( CAB \) is a horizontal straight line. The angle of elevation of \( D \) when measured from \( A \) is \( x \) degrees, and when measured from \( B \) is \( y \) degrees. The distance between \( A \) and \( B \) is \( d \) feet. Show that 
\[
CD = \frac{d \sin x \sin y}{\sin (x - y)}. \quad [10]
\]

Part III

Answer two questions from this part. Show all work.

26 Two angles of a triangle are 46° 40' and 58° 30', and the included side is 64.3 feet. Find, to the nearest tenth of a foot, the length of the shortest side of the triangle. \([10]\)

27 In triangle \( ABC \), \( a = 12, b = 18, c = 20 \).

a Find \( B \) to the nearest degree. \([5]\)

b Using the result of part a, find the area of triangle \( ABC \) to the nearest square unit. \([5]\)

28 Two forces act on an object. The first force has a magnitude of 86 pounds and makes an angle of 34° with the resultant. The magnitude of the resultant is 128 pounds. Find, to the nearest degree, the angle that the second force makes with the resultant. \([10]\)

29 Point \( B \) is N 70° E of \( A \). Point \( C \) is N 42° W of \( B \). The distance from \( C \) to \( A \) is 26 miles and the distance from \( C \) to \( B \) is 18 miles. Find, to the nearest degree, the direction of \( C \) from \( A \). \([5, 4, 1]\)
Thursday, January 22, 1959 — 9:15 a.m. to 12:15 p.m., only

Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

**Part I**

Allow 2 2/3 credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 5 is not expressed to four decimal places. For questions 13–20, allow credit if the pupil has written the correct answer instead of the letter a, b or c.

1. \( \sec 40^\circ \) or \( \csc 50^\circ \)
2. 1
3. 315
4. \( \sqrt{1 - a^2} \)
5. 0.8424
6. \( 31^\circ 11' \)
7. \( 5\sqrt{2} \)
8. \( \frac{61 - c^2}{60} \)
9. \( 2 \cos 55^\circ \sin 15^\circ \)
10. \( \frac{1}{2} \)
11. 13
12. none
13. c
14. a
15. c
16. b
17. c
18. c
19. a
20. b