Part I

Answer all questions in this part. Each correct answer will receive 2 1/2 credits.

1. Find the number of inches in the radius of a circle in which a central angle of 4 radians intercepts an arc of 6 inches. 1_____

2. What is the value of \( \cot \frac{5}{4} \)? 2_____

3. Find the positive value of \( \tan (\arcsin \frac{3}{5}) \). 3_____

4. Express \( \sin(x - y) \) in terms of the sine and cosine of \( x \) and of \( y \). 4_____

5. If \( A \) is a positive acute angle and \( \cos A = \frac{7}{9} \), find the value of \( \sin \frac{1}{2}A \). 5_____

6. In triangle \( ABC \), \( a = 5 \), \( b = 9 \) and \( \sin A = 0.30 \). Find the value of \( \sin B \). 6_____

7. In triangle \( ABC \), \( a = 4 \), \( b = 5 \) and \( c = 6 \). Find the value of \( \cos A \). 7_____

8. In triangle \( ABC \), \( a = 6 \), \( b = 4 \) and \( \tan \frac{1}{2}(A - B) = 0.50 \). Find the value of \( \tan \frac{1}{2}(A + B) \). 8_____

9. What is the area of triangle \( ABC \) if \( a = 5 \), \( b = 6 \), \( \sin C = 0.6 \), \( \cos C = 0.8 \) and \( \tan C = 0.75 \)? 9_____

10. Solve the equation \( \sin^2 \frac{1}{2}x = 1 \) for the smallest positive value of \( x \). 10_____

11. If \( \log n = 2.3571 \), find \( n \). 11_____

12. Find \( \log \sin 61^\circ 43' \). 12_____

13. Find to the nearest minute the positive acute angle whose cosine is 0.9030. 13_____

14. From an observation post 108 feet directly above a point \( A \) on level ground, the angle of depression of point \( B \) on the ground is observed to be 6° 10'. Find the distance from \( A \) to \( B \) to the nearest foot. 14_____

Directions (15-20): Indicate the correct completion for each of the following by writing on the line at the right the letter \( a \), \( b \), \( c \) or \( d \).

15. If \( \sin \theta \) is negative and \( \tan \theta \) is positive, then \( \theta \) is an angle in the (a) first quadrant (b) second quadrant (c) third quadrant (d) fourth quadrant 15_____ 

16. The function \( \csc 270^\circ \) (a) has a value of \(-1\) (b) has a value of 0 (c) has a value of 1 (d) is not defined 16_____
17. The function $\cos 250^\circ$ is equal to
   (a) $\sin 20^\circ$
   (b) $-\sin 20^\circ$
   (c) $\sin 70^\circ$
   (d) $-\sin 70^\circ$

18. If $m > 1$, the maximum value of $2m \sin 2x$ is
   (a) 2
   (b) $m$
   (c) $2m$
   (d) $4m$

19. Using the data $A = 35^\circ$, $b = 3$ and $a = 4$, it is possible to construct
   (a) two triangles
   (b) a right triangle
   (c) no triangle
   (d) an obtuse triangle

20. As $x$ varies from 45° to 315°, the graph of $y = \sin 2x$
   (a) does not cross the x-axis
   (b) crosses the x-axis once
   (c) crosses the x-axis twice
   (d) crosses the x-axis three times

Part II

Answer three questions from this part. Show all work unless otherwise directed.

21. a Starting with the formula for $\cos(x + y)$, derive the formula for $\cos 2x$ in terms of $\cos x$. [4]

   b Starting with the formula for $\cos 2x$ in terms of $\cos x$, derive the formula for $\cos \frac{1}{2}A$ in terms of $\cos A$. [6]

22. a Prove that the following equation is an identity: [5]

   \[ \frac{2 \cot x}{1 + \cot^2 x} = \sin 2x \]

   \[ \frac{\sin 7x + \sin 5x}{\cos 7x - \cos 5x} \]

   b Show that may be reduced to $-\cot x$. [5]

23. Find to the nearest degree all values of $x$ between 0° and 360°, inclusive, that satisfy the equation $3 \cos^2 x + 4 \sin x + 1 = 0$. [10]

24. a Draw the graph of $y = \tan x$ as $x$ varies from $-90^\circ$ to $+90^\circ$, assigning to $x$ the values $0^\circ$, $\pm 30^\circ$, $\pm 45^\circ$, $\pm 60^\circ$, $\pm 90^\circ$. [4]

   b On the same set of axes used in part a, draw the graph of $y = \cos x$, using the same interval and the same set of values for $x$. [4]

   c From the graphs made in answer to parts a and b, determine the number of values of $x$ that satisfy the equation $\tan x = \cos x$ when

   (1) $x$ is between $0^\circ$ and $45^\circ$. [1]

   (2) $x$ is between $45^\circ$ and $90^\circ$. [1]

25. The rectangular coordinates of the point $P$ are represented by $x$ and $y$. The distance from the origin $O$ to $P$ is represented by $r$, and the angle that $OP$ makes with the positive portion of the x-axis is represented by $\theta$.

   a Express $x$ in terms of $r$ and a trigonometric function of $\theta$. [2]

   b Express $y$ in terms of $r$ and a trigonometric function of $\theta$. [2]

   c Using the results obtained in parts a and b, show that the equation $x^2 - y^2 = 4$ can be reduced to the form $r^2 \cos 2\theta = 4$. [6]
26. In triangle $ABC$, $a = 959$, $b = 631$ and $C = 68^\circ$. Find $A$ to the nearest degree. [10]

27. Two forces of 437 pounds and 876 pounds, respectively, act upon a body at an acute angle with each other. The angle between the resultant force and the 437-pound force is $41^\circ 10'$. Find to the nearest ten minutes the angle formed by the 437-pound and the 876-pound force. [4, 6]

28. Airport $A$ is 250 miles directly west of airport $C$, and airport $B$ is 101 miles directly north of airport $C$. An airplane flies from $C$ in the direction $N \ 35^\circ \ W$ to a point $D$, which is on the direct path from $A$ to $B$. Find the distance from $C$ to $D$ to the nearest mile. [4, 6]

29. The sides of a triangle are 5.46, 6.87 and 7.65. Find the smallest angle of the triangle to the nearest degree. [10]