TRIGONOMETRY

Thursday, January 29, 1948 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 Solve the following equation for all positive values of $A$ less than $360^\circ$: $\cos 2A + \sin A = 0$. [10]

22 a By substituting $-B$ for $B$ in the formula for $\cos (A + B)$, derive the formula for $\cos (A - B)$. [3]

b Derive the law of sines for the acute plane triangle. [7]

23 In triangle $ABC$, angle $C = 90^\circ$. If angle $DAC$ is $x$ and angle $ABC$ is $2x$, prove that $d = 2c \sin x$. [10]

24 a Draw the graph of $y = \cos \frac{1}{2}x$ as $x$ varies from 0 to $2\pi$ at intervals of $\frac{\pi}{3}$. [4]

b On the set of axes used in a, draw the graph of $y = 2 \cos x$ as $x$ varies from 0 to $2\pi$ at intervals of $\frac{\pi}{3}$. [4]

c From the graphs made in answer to a and b, state the number of values of $x$ between 0 and $2\pi$ for which $\cos \frac{1}{2}x = 2 \cos x$. [2] [1]

[OVER]
Part III

Answer at least one question from part III.

28 In triangle $ABC$, $a = 32.5$, $b = 57.3$ and $c = 61.2$. Find angle $B$ to the nearest minute. [10]

26 Two straight roads meet at $P$, making an angle of $82^\circ 20'$. Camp $A$ located on one road is 238 rods from $P$. Camp $B$ on the other road is 176 rods from $P$. A direct road from $A$ to $B$ is to be constructed. At what angle with $AP$ should this road be started? [Find the angle to the nearest minute.] [10]

27 A ship is sailing on course N $40^\circ$ E. From point $A$ on its course, the bearing of lighthouse $C$ is found to be N $75^\circ$ E. When the ship has steamed 5.8 miles to point $B$, the bearing of the lighthouse from $B$ is S $62^\circ$ E. Find the distance $BC$ to the nearest tenth of a mile. [10]

Part IV

Answer at least one question from part IV.

28 In spherical triangle $ABC$, $a = 158^\circ$, $b = 90^\circ 40'$ and $c = 90^\circ$. Find angle $C$ to the nearest minute. [10]

29 An aviator flew along the great circle route from Paris (Lat. 48° 50' N, Long. 2° 20' E) to Osaka (Lat. 34° 40' N, Long. 135° 30' E). How many nautical miles did he fly? [10]
Trigonometry

Fill in the following lines:

Name of pupil..................................................Name of school..................................................

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express \( \cos 290^\circ \) as a function of a positive acute angle.
2 Express \( 105^\circ \) in radian measure. [Answer may be left in terms of \( \pi \).]
3 Find the logarithm of 3.142
4 Find \( \log \sin 63^\circ \, 48' \)
5 Find to the nearest minute the positive acute angle whose cosine is 0.8330
6 In triangle \( ABC \), \( A = 30^\circ \), \( a = 6.5 \) and \( b = 13 \). Find \( B \).
7 In triangle \( ABC \), \( a = 8 \), \( b = 13 \) and \( c = 15 \). Find \( B \).
8 In triangle \( ABC \), \( B = 105^\circ \) and \( C = 15^\circ \). Find the value of \( (b + c) : (b - c) \). [Answer may be left in radical form.]
9 If \( A \) is a positive acute angle, express \( \cot A \) in terms of \( \sin A \).
10 If \( A \) is an angle in the second quadrant and \( \sin A = \frac{5}{8} \), find \( \sin 2A \).
11 If \( x \) is an acute angle and \( \cos x = \frac{1}{3} \), find \( \cos \frac{x}{2} \).
12 Express \( \sin 3A + \sin A \) as a product of two functions.

Directions (questions 13–17) — Indicate the correct answer to each question by writing the letter \( a \), \( b \) or \( c \) on the line at the right.

13 As an angle increases from \( \pi \) to \( 2\pi \), its cosine (a) increases throughout (b) decreases and then increases (c) increases and then decreases

14 The value of \( \sin^2 2A + \cos^2 2A \) is (a) 1 (b) 2 (c) 4

15 From the data \( B = 35^\circ \), \( b = 6 \) and \( a = 10 \), there can be constructed (a) no triangle (b) only one triangle (c) two triangles

16 If in spherical triangle \( ABC \), \( C = 90^\circ \), \( e = 70^\circ \) and \( b = 50^\circ \), then \( a \) is (a) greater than 90° (b) equal to 90° (c) less than 90°

17 The statement \( \cos 2x = (\cos x - \sin x)(\cos x + \sin x) \) is (a) true for all values of \( x \) (b) true for only certain values of \( x \) (c) not true for any value of \( x \)

18 Find to the nearest integer, the area of triangle \( ABC \) in which \( a = 12 \), \( c = 15 \) and \( B = 18^\circ \).

19 Find the value of \( x \) greater than 0° and less than 90° which satisfies the equation \( \sin 2x = \cos x \).

20 In right spherical triangle \( ABC \) in which \( C \) is the right angle, \( a \) and \( b \) are known. Write a formula that can be used to find \( A \).