

TRIGONOMETRY

Thursday, January 30, 1947 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 a Find, correct to the nearest minute, the positive acute angle which satisfies the equation $3 \cos^2 x + 4 \sin x - 4 = 0$ [8]

b How many angles between 0° and 360° , including the one found in answer to a, satisfy the given equation? [2]

22 a Derive the law of cosines for the acute plane triangle. [8]

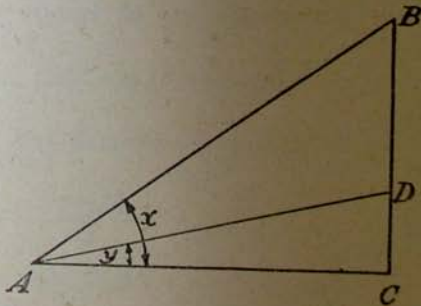
b Starting with the formula for $\sin(x + y)$, derive the formula for $\sin 2x$. [2]

23 In triangle ABC , $\angle C = 90^\circ$

Let $\angle BAC$ be represented by x and

$\angle DAC$ be represented by y . Prove that

$$BD = \frac{AD(\sin x \cos y - \cos x \sin y)}{\cos x} \quad [10]$$



24 a Draw the graph of $y = \sin \frac{1}{2} x$ as x varies from 0° to 360° at intervals of 60° [8]

b On the same set of axes used in answering a, draw the graph of $y = .6$ [1]

c By means of the graphs drawn in answer to a and b, determine the number of values of x between 0° and 360° for which $\sin \frac{1}{2} x = .6$ [1]

[1]

[OVER]

Part III

Answer at least one question from part III.

25 In triangle ABC , $A = 70^\circ$, $C = 66^\circ$ and $AB = 120$ a Find AC correct to the nearest tenth. [6]b Find, correct to the nearest integer, the area of triangle ABC . [4]

26 A tower 45 feet high stands on level ground. Two points, A and B , are on the same level with the foot of the tower and directly east of it. From the top of the tower the angles of depression of A and B are 19° and 15° respectively. Find the distance from A to B . [10]

27 Two vessels leave a port at the same time. One sails $N 62^\circ 50' E$ at the rate of 17 miles per hour and the other sails $S 52^\circ E$ at the rate of 21 miles per hour. How far apart are the ships 1 hour after leaving port? Give answer correct to the nearest mile. [10]

Part IV

Answer at least one question from part IV.

28 Given right spherical triangle ABC in which C is the right angle. If $b = 108^\circ$ and $A = 34^\circ$, find B correct to the nearest minute. [10]

29 Find, correct to the nearest nautical mile, the distance from New York (Lat. $40^\circ 46' N$, Long. $73^\circ 58' W$) to Honolulu (Lat. $21^\circ 18' N$, Long. $157^\circ 52' W$). [10]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 Express $\cot 190^\circ$ as a function of a positive angle less than 45° . 1.....
- 2 Express 250° in radian measure. [Answer may be left in terms of π .] 2.....
- 3 Find the logarithm of 0.8926 3.....
- 4 Find $\log \tan 36^\circ 28'$ 4.....
- 5 Find the value of $\sin 33^\circ 18'$ 5.....
- 6 Find, correct to the nearest minute, the positive acute angle whose cosine is 0.8613 6.....
- 7 Find the positive acute angle which satisfies the equation $4 \cos^2 x = 1$ 7.....
- 8 In triangle ABC , $a = 8$, $b = 7$ and $c = 4$. Find $\cos A$. 8.....
- 9 In triangle ABC , $A = 30^\circ$, $B = 45^\circ$ and $a = 3$. Find b . [Answer may be left in radical form.] 9.....
- 10 In triangle ABC , $a = 5$, $b = 3$ and $C = 60^\circ$. Find $\tan \frac{1}{2}(A - B)$. [Answer may be left in radical form.] 10.....
- 11 If $\cos x = a$, express $\cos^2 \frac{x}{2}$ in terms of a . 11.....
- 12 Complete the formula $\cos(x - y) = \dots$ 12.....
- 13 Express the area of a triangle in terms of its sides a , b and c and its semiperimeter s . 13.....
- 14 If A is an angle in the first quadrant, express $\cot A$ in terms of $\cos A$. 14.....
- 15 In right spherical triangle ABC in which C is the right angle, a and c are known. Write the formula that should be used to find b . 15.....
- 16 One angle of a spherical triangle is 110° . Find the side of the polar triangle which lies opposite this angle. 16.....

Directions (questions 17-20) — Indicate the correct answer to each question by writing the letter a , b or c on the line at the right.

- 17 The plane triangle in which $a = 85$, $c = 65$ and $A = 123^\circ$ has (a) one solution (b) two solutions (c) no solution 17.....
- 18 As angle A increases from 90° to 180° , $\cos A$ (a) decreases from 1 to 0 (b) increases from -1 to 0 (c) decreases from 0 to -1 18.....
- 19 The value of $\sin \frac{\pi}{3}$ is (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{-\sqrt{3}}{2}$ 19.....
- 20 In right spherical triangle ABC in which C is the right angle, if $a = 61^\circ$ and $b = 144^\circ$, then c is (a) greater than 90° (b) less than 90° (c) equal to 90° 20.....