

The University of the State of New York

296TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, January 31, 1946—9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II, III and IV, including at least one question from each part.

Part II

Answer at least one question from part II.

21 a Starting with the formulas for  $\sin(x + y)$  and  $\cos(x + y)$ , derive the formula for  $\tan(x + y)$ . [6]

b Express  $\tan(135^\circ + \theta)$  in terms of  $\tan \theta$ . [4]

22  $A$  and  $C$  are angles of an acute plane triangle,  $b$  is the included side,  $h$  is the altitude upon  $b$ , and  $K$  is the area of the triangle.

a Show that  $h$  is given by the formula  $h = \frac{b \sin A \sin C}{\sin(A + C)}$  [8]

b Show that  $K$  is given by the formula  $K = \frac{b^2 \sin A \sin C}{2 \sin(A + C)}$  [2]

23 a Draw and letter clearly the line values of the sine, cosine, and tangent of an angle in the third quadrant. [1, 1, 2]

b (1) Sketch on one set of axes the graphs of  $\cos x$  and  $\cos 2x$  as  $x$  varies from 0 to  $2\pi$  radians. [2, 3]

(2) By means of these graphs show that there are just two values of  $x$  greater than 0 and less than  $2\pi$  which satisfy the equation  $\cos x = \cos 2x$  [1]

24 Find, correct to the nearest minute, the positive acute angle which satisfies the equation  $2 \sec^2 x + \tan x = 3$  [10]

## Part III

Answer at least one question from part III.

- 25 In triangle  $ABC$ ,  $A = 40^\circ 20'$ ,  $a = 9$ , and  $c = 13$ . Find  $C$  correct to the nearest minute. [10]
- 26 Point  $B$  is 450 rods directly east of point  $A$ . The bearing of point  $C$  from  $B$  is  $N 20^\circ W$  and point  $C$  is 670 rods from  $B$ . Find, correct to the nearest minute, the bearing of  $C$  from  $A$ . [10]
- 27 A railroad runs from point  $A$  directly north to point  $B$ , a distance of 60 miles. An enemy gun is located east of the railroad, 30 miles from  $A$  and 40 miles from  $B$  and has a range of 19 miles. Is the railroad within range of the gun? [All computation in this problem must be shown.] [10]

## Part IV

Answer at least one question from part IV.

- 28 In the isosceles spherical triangle  $ABC$ ,  $AC = CB$ ,  $A = 70^\circ$  and  $C = 100^\circ$ . Find  $AC$ , correct to the nearest minute. [10]
- 29 The great circle arc between Tokyo and Wake Island is  $28^\circ 45'$  and the bearing of Tokyo from Wake Island is  $N 49^\circ 43' W$ . The longitude of Tokyo is  $139^\circ 45' E$  and the longitude of Wake Island is  $166^\circ 35' E$ . Find the latitude of Tokyo. [10]

## Fill in the following lines:

Name of school.....Name of pupil.....

## Part I

Answer all questions in part I. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 Find, correct to the nearest minute, the acute angle whose sine is .9618 1.....
- 2 Find  $\log \cos 29^\circ 46'$  2.....
- 3 Point  $A$  is 200 miles from  $B$  and in the direction  $S 18^\circ E$  from  $B$ . How far is  $A$  east of  $B$ ? [Give your answer correct to the nearest mile.] 3.....
- 4 Two forces, one of 100 pounds and the other of 40 pounds, act upon a body at an angle of  $90^\circ$ . Find, correct to the nearest degree, the angle between the larger force and the resultant of the two forces. 4.....
- 5 If  $\sin x = a$  and  $\cos x = b$ , express  $\sin 2x$  in terms of  $a$  and  $b$ . 5.....
- Directions (questions 6-9) — Indicate the correct answer to each question by writing on the line at the right the letter  $a$ ,  $b$  or  $c$ .
- 6 The value of  $\sin \frac{11\pi}{6}$  is (a)  $\frac{1}{2}$  (b)  $-\frac{1}{2}$  (c) neither of these two values 6.....
- 7 As  $x$  increases from  $0^\circ$  to  $90^\circ$ , the function  $\sin x + \cos x$  (a) increases and then decreases (b) decreases and then increases (c) remains the same 7.....
- 8 The statement  $\sin^2 A - \cos^2 A = 1$  is (a) true for all values of  $A$  (b) true for only certain values of  $A$  (c) not true for any value of  $A$  8.....
- 9 The statement  $\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$  is (a) true for all values of  $\theta$  (b) true for only certain values of  $\theta$  (c) not true for any value of  $\theta$  9.....
- 10 Is the following statement true or is it false?  
If  $A$  is an angle in the second quadrant, its tangent is negative. 10.....
- 11 Express  $\cos (360^\circ - x)$  in terms of  $\cos x$ . 11.....
- 12 If  $x$  and  $y$  are positive acute angles and if  $\sin x = \frac{4}{5}$  and  $\sin y = \frac{3}{5}$ , find the value of  $\sin (x - y)$ . 12.....
- 13 Express  $\cos 40^\circ + \cos 10^\circ$  in terms of the product of two cosines. 13.....
- 14 In triangle  $ABC$ ,  $C = 60^\circ$ . Using the law of cosines for plane triangles, express  $c$  in terms of  $a$  and  $b$ . 14.....
- 15 In plane triangle  $ABC$ ,  $B = 60^\circ$  and  $C = 45^\circ$ . Find the value of the ratio of  $b$  to  $c$ . [Answer may be left in radical form.] 15.....
- 16 In triangle  $ABC$ ,  $a = 12$ ,  $b = 8$  and  $C = 90^\circ$ . Find the value of  $\tan \frac{1}{2}(A - B)$ . 16.....
- 17 Find, correct to the nearest integer, the number of minutes in an angle of 10 mils. 17.....
- 18 Is the following statement true or is it false?  
If the side opposite the right angle of a right spherical triangle is greater than  $90^\circ$ , then both of the sides including the right angle are greater than  $90^\circ$ . 18.....
- 19 Is a right spherical triangle determined if an angle and the side opposite are given? [Answer yes or no.] 19.....
- 20 In right spherical triangle  $ABC$  in which  $C$  is the right angle,  $b$  and  $a$  are known. Write the formula that should be used to find  $A$ . 20.....