The University of the State of New York
286th High School Examination

TRIGONOMETRY

Thursday, January 21, 1943 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 Answer a and either b or c:

a Find, correct to the nearest degree, the positive acute angle which satisfies the equation

\[ 3 \cos^2 x + 7 \sin x - 5 = 0. \]  [5]

b Prove the identity

\[ \frac{2 \sin^2 x}{\sin 2x} + \cot x = \sec x \csc x. \]  [5]

* If in the spherical triangle \( ABC \) angle \( C \) is a right angle and \( a \) and \( A \) are given, write the formula that should be used to find \( b \).  [5]

22 a Starting with the formula for \( \cos (x + y) \), derive the formula for \( \cos 2x \) in terms of \( \cos x \).  [5]

b In right triangle \( ABC \), \( C \) is the right angle. Prove: \( \tan \frac{1}{2} A = \sqrt{\frac{c - b}{c + b}} \)  [5]

23 A lighthouse whose height is \( h \) stands on a rock. A ship is sighted. From the bottom of the lighthouse the angle of depression of the ship is \( A \); from the top of the lighthouse the angle of depression is \( B \). Derive a formula for the horizontal distance \( d \) between the ship and the lighthouse in terms of \( h \), \( A \) and \( B \).  [10]

24 a Draw the graph of \( y = \sin 2x \) as \( x \) varies from \( 0^\circ \) to \( 180^\circ \) inclusive at intervals of \( 15^\circ \).  [4]

b Using the same set of axes as in a, draw the graph of \( y = \tan x \), as \( x \) varies from \( 0^\circ \) to \( 180^\circ \) inclusive at intervals of \( 30^\circ \).  [4]

c With the aid of the graphs drawn in answer to a and b, find all the values of \( x \) between \( 0^\circ \) and \( 180^\circ \) for which \( \sin 2x = \tan x \).  [2]

* This question is based on new material found in the revised syllabus.
25 In $\triangle ABC$, $a = 32.2$, $b = 43.3$, $c = 55.5$. Find $C$, correct to the nearest minute. \[10\]

26 Stations $A$ and $B$ on a straight horizontal railroad are 5280 feet apart. An airplane $P$ is flying directly above the railroad. At a certain moment the angles of elevation $PAB$ and $PBA$, as observed from the stations, are $38^\circ 30'$ and $63^\circ 40'$. Find, correct to the nearest 10 feet, the height of the airplane above the railroad. \[10\]

27 Given a quadrangle $ABCD$ in which $AB = 60$, $BC = 80$ and $CD = 90$. Angle $B$ is $90^\circ$ and diagonal $AC$ makes an angle of $50^\circ$ with $CD$. Find, correct to the nearest integer, the area of $ABCD$. \[10\]

28 A gun fired at $A$ was heard at $B$ and at $C$ two seconds and three seconds respectively after it was fired. If angle $BAC = 110^\circ 30'$ and the sound traveled 1150 feet per second, compute, correct to the nearest foot, the distance between $B$ and $C$. \[10\]

*29 a State the Law of Sines for oblique spherical triangles. \[3\]

$\quad b$ In spherical triangle $ABC$, $A = 110^\circ$, $a = 147^\circ$ and $B = 133^\circ$; using the Law of Sines, find $b$ correct to the nearest degree. \[7\]

* This question is based on new material found in the revised syllabus.
Trigonometry

Fill in the following lines:

Name of school........................................Name of pupil........................................

Part I

Answer all questions in this part. Each correct answer will receive \(2\frac{1}{2}\) credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express \(\frac{2\pi}{3}\) radians in degrees.

2 Express in degrees the positive acute angle \(A\), if \(\sin A = \cos 34^\circ\)

3 Express sin \(297^\circ\) as a function of a positive acute angle.

4 Find, correct to the nearest minute, the positive acute angle \(A\), if \(\cos A = 0.9830\)

5 If \(\log \sin A = 9.3692 - 10\) and \(A\) is a positive acute angle, find the value of \(A\) correct to the nearest minute.

6 Find the value of \(\tan 31^\circ 24^\prime\)

7 Find, correct to the nearest integer, the number whose logarithm is 3.8558

8 Find the positive acute angle that satisfies the equation \(3 \tan^2 x - 1 = 0\)

9 In \(\triangle ABC\), \(a = 5\), \(b = 8\), \(c = 7\); find \(C\).

10 Given \(a\), \(b\) and \(C\) in \(\triangle ABC\); complete the formula \(\tan \frac{A}{2} (A - B) = \ldots\) that would be used in finding \(A\) and \(B\).

11 If \(\cos x = a\), express \(\tan^2 \frac{x}{2}\) in terms of \(a\).

12 An artillery observer in a captive balloon 3000 feet directly above his guns finds that the angle of depression of an enemy's machine-gun nest is \(26^\circ 20^\prime\). Compute, correct to the nearest foot, the distance between the observer's guns and the enemy's nest. [Assume that both lie at the same elevation.]

13 Express \(\sin 130^\circ + \sin 10^\circ\) as a function of \(70^\circ\)

14 For how many values of \(x\) between \(0^\circ\) and \(180^\circ\) does the line \(y = \frac{1}{2}\) intersect the curve \(y = \sin x\)?

Directions (questions 15–20) — Indicate the correct answer to each question by writing on the line at the right the letter \(a\), \(b\) or \(c\).

15 For all values of angle \(x\), \((a)\) \(\sin (-x) = \sin x\), \((b)\) \(\cos (-x) = \cos x\) or \((c)\) \(\tan (-x) = \tan x\).

16 \(\tan (45^\circ + x)\) is equal to \((a)\) \(1 + \tan x\), \((b)\) \(\frac{1 - \tan x}{1 + \tan x}\) or \((c)\) \(\frac{1 + \tan x}{1 - \tan x}\)

17 As \(A\) increases from \(270^\circ\) to \(360^\circ\), \((a)\) \(\sin A\) increases from \(0\) to \(1\), \((b)\) \(\cos A\) increases from \(0\) to \(1\) or \((c)\) \(\tan A\) increases from \(-\infty\) to \(+\infty\)

[3]
18 The number of different triangles that can be formed in which \( A = 48^\circ \), 
\( a = 50 \) and \( b = 64 \), is (a) two, (b) one or (c) none.

19 The maximum value of \( 2 \sin 2A \) is (a) 1, (b) 2 or (c) 4

20 \( \log \sqrt{\frac{a}{b}} \) equals (a) \( \sqrt{\log a - \log b} \), (b) \( \frac{1}{2} \times \frac{\log a}{\log b} \) or 
(c) \( \frac{1}{2} (\log a - \log b) \)