The University of the State of New York

286TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, January 21, 1943 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 Answer *a* and *either* *b* or *c*:

a Find, correct to the *nearest degree,* the positive acute angle which satisfies the equation $3\cos^2 x + 7\sin x - 5 = 0$. [5]

b Prove the identity $\dfrac{2\sin^2 x}{\sin 2x} + \cot x = \sec x \csc x$. [5]

**c* If in the spherical triangle ABC angle C is a right angle and a and A are given, write the formula that should be used to find b. [5]

22 *a* Starting with the formula for $\cos(x + y)$, derive the formula for $\cos 2x$ in terms of $\cos x$. [5]

b In right triangle ABC, C is the right angle. Prove: $\tan \frac{1}{2} A = \sqrt{\dfrac{c - b}{c + b}}$ [5]

23 A lighthouse whose height is h stands on a rock. A ship is sighted. From the bottom of the lighthouse the angle of depression of the ship is A; from the top of the lighthouse the angle of depression is B. Derive a formula for the horizontal distance d between the ship and the lighthouse in terms of h, A and B. [10]

24 *a* Draw the graph of $y = \sin 2x$ as x varies from 0° to 180° inclusive at intervals of 15°. [4]

b Using the same set of axes as in *a*, draw the graph of $y = \tan x$, as x varies from 0° to 180° inclusive at intervals of 30°. [4]

c With the aid of the graphs drawn in answer to *a* and *b*, find all the values of x between 0° and 180° for which $\sin 2x = \tan x$. [2]

* This question is based on new material found in the revised syllabus.

Part III

Answer at least two questions from part III.

25 In $\triangle ABC$, $a = 32.2$, $b = 43.3$, $c = 55.5$. Find C, correct to the *nearest minute.* [10]

26 Stations A and B on a straight horizontal railroad are 5280 feet apart. An airplane P is flying directly above the railroad. At a certain moment the angles of elevation PAB and PBA, as observed from the stations, are 38° 30′ and 63° 40′. Find, correct to the *nearest 10 feet,* the height of the airplane above the railroad. [10]

27 Given a quadrangle $ABCD$ in which $AB = 60$, $BC = 80$ and $CD = 90$. Angle B is 90° and diagonal AC makes an angle of 50° with CD. Find, correct to the *nearest integer,* the area of $ABCD$. [10]

28 A gun fired at A was heard at B and at C two seconds and three seconds respectively after it was fired. If angle $BAC = 110° \; 30'$ and the sound traveled 1150 feet per second, compute, correct to the *nearest foot,* the distance between B and C. [10]

*29 *a* State the Law of Sines for oblique spherical triangles. [3]
b In spherical triangle ABC, $A = 110°$, $a = 147°$ and $B = 133°$; using the Law of Sines, find b correct to the *nearest degree.* [7]

* This question is based on new material found in the revised syllabus.

172

Fill in the following lines:

Name of school.......................................Name of pupil.......................................

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express $\frac{2\pi}{3}$ radians in degrees. 1................

2 Express in degrees the positive acute angle A, if $\sin A = \cos 34°$ 2................

3 Express $\sin 297°$ as a function of a positive acute angle. 3................

4 Find, correct to the *nearest minute*, the positive acute angle A, if $\cos A = 0.9830$ 4................

5 If $\log \sin A = 9.3692 - 10$ and A is a positive acute angle, find the value of A correct to the *nearest minute*. 5................

6 Find the value of $\tan 31° 24'$ 6................

7 Find, correct to the *nearest integer*, the number whose logarithm is 3.8558 7................

8 Find the positive acute angle that satisfies the equation $3 \tan^2 x - 1 = 0$ 8................

9 In $\triangle ABC$, $a = 5$, $b = 8$, $c = 7$; find C. 9................

10 Given a, b and C in $\triangle ABC$; complete the formula $\tan \frac{1}{2}(A - B) = \ldots$ that would be used in finding A and B. 10................

11 If $\cos x = a$, express $\tan^2 \frac{x}{2}$ in terms of a. 11................

12 An artillery observer in a captive balloon 3000 feet directly above his guns finds that the angle of depression of an enemy's machine-gun nest is 26° 20′. Compute, correct to the *nearest foot*, the distance between the observer's guns and the enemy's nest. [Assume that both lie at the same elevation.] 12................

13 Express $\sin 130° + \sin 10°$ as a function of 70° 13................

14 For how many values of x between 0° and 180° does the line $y = \frac{1}{2}$ intersect the curve $y = \sin x$? 14................

Directions (questions 15–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

15 For all values of angle x, (*a*) $\sin(-x) = \sin x$, (*b*) $\cos(-x) = \cos x$ or (*c*) $\tan(-x) = \tan x$. 15......

16 Tan $(45° + x)$ is equal to (*a*) $1 + \tan x$, (*b*) $\frac{1 - \tan x}{1 + \tan x}$ or (*c*) $\frac{1 + \tan x}{1 - \tan x}$ 16......

17 As A increases from 270° to 360°, (*a*) $\sin A$ increases from 0 to 1, (*b*) $\cos A$ increases from 0 to 1 or (*c*) $\tan A$ increases from $-\infty$ to $+\infty$ 17......

173

18 The number of different triangles that can be formed in which $A = 48°$, $a = 80$ and $b = 64$, is (*a*) two, (*b*) one or (*c*) none. 18......

19 The maximum value of $2 \sin 2A$ is (*a*) 1, (*b*) 2 or (*c*) 4 19......

20 Log $\sqrt{\frac{a}{b}}$ equals (*a*) $\sqrt{\log a - \log b}$, (*b*) $\frac{1}{2} \times \frac{\log a}{\log b}$ or (*c*) $\frac{1}{2}(\log a - \log b)$ 20......