Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement in either plane trigonometry or spheric trigonometry is one recitation a week for a school year or two recitations a week for half a school year.

To receive credit for plane trigonometry students should answer three questions from group I and three questions from group II.
To receive credit for both plane and spheric trigonometry students should answer three questions from group I and three questions from group III.

Group I

1. Calculate by means of logarithms \( \sqrt[3]{34.71 \times 0.002098} \)

2. Show by means of a geometric diagram or otherwise that

\[
\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}
\]

3. Prove that \( \sec^2 x + \csc^2 x = \sec^2 x \csc^2 x \)

4. Find the values of \( x \) between \( 0^\circ \) and \( 360^\circ \) that will satisfy the equation \( \cos 2x(3 - 4 \cos^2 x) = 0 \)

Group II

5. Prove \( \frac{a + b}{c} = \frac{\cos \frac{1}{2}(A - B)}{\sin \frac{1}{2} C} \)

6. State the law of cosines and give proof.

7. An observer in a balloon measures the angle of depression of an object on the ground and finds it to be \( 63^\circ \ 58' \); after ascending vertically 582 feet, he finds that the angle of depression of the same object is \( 74^\circ \ 49' \). What was the height of the balloon at the time of the first observation?

8. Given \( a = 168.32, b = 221.46, A = 33^\circ \ 39' \ 16'' \); find \( B \).

Group III

9. State Napier's rules for right spheric triangles. By means of these rules write down four formulas that may be used in the solution of right spheric triangles and prove one of them.

10. In solving oblique spheric triangles what are the conditions for two solutions? Demonstrate.

11. In a spheric triangle given \( a = 51^\circ \ 43' \ 18'', b = 38^\circ \ 2' \ 20'', c = 75^\circ \ 11' \ 30'' \); find \( A \).

12. In a right spheric triangle given \( a = 46^\circ \ 50', b = 31^\circ \ 15' \); find \( c \) and \( A \).