Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since no credit will be given any answer in group I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in mathematics third year.

The minimum time requirement is five recitations a week for a school year after the completion of elementary algebra.

In this examination the customary lettering is used. $A$, $B$ and $C$ represent the angles of a triangle $ABC$; $a$, $b$ and $c$ represent the respective opposite sides. In a right triangle, $C$ represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Answer five questions from these two groups, including at least two questions from each group.
Name of school........................................Name of pupil........................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial
credit will be allowed. Each answer must be reduced to its simplest form.

1. Express 300° in radians. [Answer may be left in terms of π.]

2. The formula for the last term of a geometric progression in terms of the first term a, the common ratio r and the number of terms n, is \( l = \ldots \).

3. Find the number of degrees in the acute angle \( x \), if \( x = \sin^{-1} \frac{1}{2} \).

4. Find the positive acute angle \( A \) correct to the nearest minute, if \( \log \sin A = 9.7062 - 10 \).

5. Find \( \log \cos 42° 17' \).

6. If \( A = \pi r^2 \), express \( \log A \) in terms of \( \log \pi \) and \( \log r \).

7. Express \( \frac{1}{\sec^2 A} \) as a function of \( \sin A \).

8. Express tan \( (-345°) \) as a function of a positive acute angle less than 45°.

9. The roots of the equation \( 2x^2 - x + 7 = 0 \) are \( (a) \) real, equal and rational, \( (b) \) real, unequal and rational, \( (c) \) real, unequal and irrational or \( (d) \) imaginary. Which is correct \( (a) \), \( (b) \), \( (c) \) or \( (d) \)?

10. What is the sum of the roots of the equation \( 2x^2 - 7x + q = 0 \)?

11. Find both values of \( \tan A \) if \( \sqrt{\tan^2 A + 16} = 5 \).

12. Find the positive value of \( \cos A \) if \( 3 \cos^2 A + 2 \cos A = 1 = 0 \).

13. Find the value of \( 8\sec^2 A \).

14. Find the second term of the expansion of \( (x^2 + 1)^3 \).

15. Write the equation of the straight line which passes through the points whose coordinates are given in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1</td>
<td>3</td>
<td>7</td>
<td>11</td>
<td>15</td>
</tr>
</tbody>
</table>

16. In the triangle \( ABC \), find \( b \) if \( \sin B = .3 \), \( \sin C = .2 \) and \( c = 1.8 \).

17. In triangle \( ABC \), \( a = 3 \), \( c = 4 \), and \( \cos B = \frac{3}{4} \); find \( b \).

18. The graph of \( y = \cos x \) for the range 0° to 360° intersects the graph of \( y = .3 \) in \( (a) \) one point, \( (b) \) two points, \( (c) \) four points. Which is correct \( (a) \), \( (b) \) or \( (c) \)?

19. The graphical solution of the set of simultaneous equations \( x^2 + y^2 = 40 \) and \( xy = 12 \) requires finding the coordinates of the points of intersection of \( (a) \) a circle and a straight line, \( (b) \) a circle and a hyperbola, \( (c) \) a hyperbola and a straight line, \( (d) \) an ellipse and a parabola. Which is correct \( (a) \), \( (b) \), \( (c) \) or \( (d) \)?

20. Find the acute angle which the graph of the equation \( y = x - 5 \) forms with the \( x \) axis.
Mathematics — Third Year

See instructions for groups II and III on page 1.

Answer five questions from groups II and III, including at least two questions from each group.

Group II

Answer at least two questions from this group.

21 Find, correct to the nearest tenth, the roots of the equation \(2x^2 + 5x - 4 = 0\) \[10\]

22 A number consists of two digits. The tens digit exceeds twice the units digit by 1. If the digits are reversed, the resulting number is 36 less than the original number. Find the number. \[6, 4\]

23 A dealer bought a certain number of books of the same kind for $60. He presented 4 copies to friends as gifts and sold the rest for $90 at an advance of $1 per book. How many books did he buy? \[6, 4\]

24 A man agrees to contribute 10 cents each week toward his son’s savings provided his son saves at the following rate: 5 cents the first week, 7 cents the second week, 9 cents the third week, etc. After how many weeks will the son’s savings together with the father’s contributions amount to $2.75? [Solution by arithmetic will not be accepted.] \[10\]

25 a Construct the graph of the equation \(y = x^2 - 3\) from \(x = -3\) to \(x = +3\) inclusive. \[6\]

b On the same set of axes used in answer to a, draw a circle having its center at the origin and its radius 4. \[1\]

c Write the equation of the circle constructed in answer to b. \[1\]

d From the graphs made in answer to a and b estimate, correct to the nearest tenth, the coordinates of the points of intersection. \[2\]

*26 The depth of a small rectangular box is 2 inches more than the width and the length exceeds the width by 7 inches. If the capacity of the box is 624 cubic inches, find its dimensions. \[4, 6\]

Group III

Answer at least two questions from this group.

27 At a point \(A\) the angle of elevation of the top of a hill is \(19^\circ 21'\); at a point \(B\) in the same horizontal plane and 1100 feet nearer the foot of the hill, the angle of elevation is \(34^\circ 40'\). How far is point \(B\) from the top of the hill? [Express answer correct to the nearest foot.] \[10\]

28 In triangle \(ABC\), \(AB = 23.5\), \(BC = 30.2\), and \(AC = 34.3\); using logarithms, find angle \(A\) correct to the nearest minute. \[10\]

29 Prove that \(\sin(x + y) = \sin x \cos y + \cos x \sin y\) in which \(x\), \(y\) and \((x + y)\) are positive acute angles. \[10\]

* This question is based on one of the optional topics in the syllabus.