Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since no credit will be given any answer in group I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in mathematics third year.

The minimum time requirement is five recitations a week for a school year after the completion of elementary algebra.

In this examination the customary lettering is used. $A$, $B$ and $C$ represent the angles of a triangle $ABC$; $a$, $b$ and $c$ represent the respective opposite sides. In a right triangle, $C$ represents the right angle.

Give special attention to neatness and arrangement of work.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.
Fill in the following lines:

Name of school...........................................................................Name of pupil.................................................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2\(\frac{1}{2}\) credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Find the logarithm of .2834  
2 If \(\sin A = .5123\), find the acute angle \(A\) correct to the nearest minute.  
3 If \(\log \cot B = 9.3798 - 10\), find the acute angle \(B\) correct to the nearest minute.  
4 The roots of \(2x^2 - 8x - 9 = 0\) are \((a)\) imaginary; \((b)\) real, equal and rational; \((c)\) real, unequal and irrational or \((d)\) real, unequal and rational. Which is correct, \((a)\), \((b)\), \((c)\) or \((d)\)?  
5 Express \(\sqrt{-18}\) in terms of the imaginary unit \(i\).  
6 Find the value of \(5^6 + 16^4 - \sqrt{34}\).  
7 Simplify \(\left(\frac{b}{a-b} + 1\right) \div \frac{a}{b-a}\).  
8 Form the quadratic equation whose roots are \(-5\) and \(+8\) .  
9 Express the fraction \(\frac{7}{\sqrt{5} - \sqrt{2}}\) with a rational denominator.  
10 Insert two geometric means between 54 and 2 .

11 In the formula \(K = \frac{7}{3 + \frac{1}{b}}\), when \(b\) is positive, does \(K\) increase or decrease as \(b\) increases?  
12 Given the formula \(\frac{1}{N} = \frac{1}{C} + \frac{1}{R}\); express \(C\) as a function of \(R\) and \(N\).  
13 The sine of an angle is the reciprocal of \((a)\) the cosine, \((b)\) the cosecant or \((c)\) the secant. Which is correct, \((a)\), \((b)\) or \((c)\)?  
14 Express \(105^\circ\) in radian measure. [Leave answer in terms of \(\pi\).]  
15 In which quadrant is an angle if its cosine is negative and its tangent is positive?  
16 Express \(\tan 130^\circ\) as a function of a positive angle less than \(45^\circ\).  
17 Solve the following equation for the positive acute value of \(x\): \(2 \sin^2 x + \sin x - 1 = 0\).  
18 In triangle \(ABC\), \(b = 6\), \(c = 5\), \(\cos A = \frac{1}{4}\); find \(a\). [Answer may be left in radical form.]  
19 What is the value of the tangent of the angle which the line \(y = 3x - 7\) makes with the \(x\)-axis?  
20 If \(x = \sin^{-1} \left(\frac{\sqrt{3}}{2}\right)\) and \(x\) is acute, find \(\tan x\).
21 Solve the following pair of simultaneous equations; give the values of \( x \) and \( y \) correct to the nearest tenth:
\[
\begin{align*}
x^2 + y^2 &= 6 \\
y &= x + 1
\end{align*}
\]

22 In an arithmetic progression \( a \) is the first term, \( d \) the common difference, \( n \) the number of terms, \( l \) the last term and \( S \) the sum.
   a Derive the formula for \( S \) in terms of \( a, n \) and \( l \). \[7\]
   b Using this formula and the formula for \( l \), derive the formula for \( S \) in terms of \( a, n \) and \( d \). \[3\]

23 A real-estate agent bought a number of acres of land for $900. He kept 10 acres of land for himself and sold the remainder at an advance of $10 an acre. If he received $1050 for the land he sold, how many acres did he buy? \[7, 3\]

24 A and B start at the same time from the same point and travel on roads that are at right angles to each other. \( A \) travels 4 miles per hour faster than \( B \). At the end of two hours they are 40 miles apart. Find the rate of each. \[7, 3\]

25 a Plot the graph of \( x^2 - 3x = y \) from \( x = -2 \) to \( x = 5 \) inclusive. \[6\]
   b Write the equation of the axis of symmetry of the curve made in answer to a. \[2\]
   c Using the graph, estimate the roots of \( x^2 - 3x = 1 \) correct to the nearest tenth. \[2\]

*26 Solve for \( x \) correct to the nearest hundredth:
\[
(3.2)^{x-1} = 4 \quad [10]
\]

**Group III**

Answer two questions from this group.

27 A tower stands on a horizontal plane. From a point in this plane the angle of elevation of the top of the tower is \( 62^\circ \); from a point 50 feet farther from the foot of the tower, the angle of elevation is \( 48^\circ \). Find the height of the tower correct to the nearest foot. \[10\]

28 In triangle \( ABC \), \( a = 150 \), angle \( B = 84^\circ \), \( c = 96 \); find \( A \) correct to the nearest minute. \[10\]

29 a Solve for all positive values of \( x \) between \( 0^\circ \) and \( 180^\circ \) that satisfy the equation
\[
\cos^2 x = 2 \sin^2 x + \frac{1}{2} \sin x \quad [5]
\]
   b Prove the identity: \( \cot A - \tan A = 2 \cot 2A \) \[5\]

* This question is based on one of the optional topics in the syllabus.