Instructions

Do not open this sheet until the signal is given.
Answer all questions in part I and five questions from part II.
Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and reduced to its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.
In both parts of this examination the use of the slide rule will be allowed for checking; in part II all computations with tables must be shown on the answer paper.
Fill in the following lines:

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each question has 2½ credits assigned to it; no partial
credit should be allowed. Each answer must be reduced to its simplest form.

1 Express $210^\circ$ in radians.

2 Find the value of $\tan (-135^\circ)$

3 What curve is represented by the equation
   \[9x^2 + 16y^2 = 144?\]

4 Solve for $h$ the formula $r = \frac{3v}{\pi h}$

5 Write the first three terms of the expansion $(1 - y)^5$

6 Find $\log \cos 25^\circ 43'$

7 Find, correct to the nearest minute, the smallest positive value of $A$ if $\log \sin A = 9.8673 - 10$

8 Find, correct to the nearest foot, the height of a flagpole if the angle of
elevation of the top is $27^\circ 40'$ at a point 100 feet from the base.

9 In a triangle $a = 20$ inches, $c = 45$ inches and $B = 47^\circ$; find the
   area of the triangle correct to the nearest square inch.

10 Find the three factors of $9 \cos^3 x - \cos x$

11 To reduce $\frac{\sin 2x}{\sin x}$ to lowest terms, by what must the numerator and
denominator be divided?

12 Find the value of $8^{-\frac{3}{2}} \times 3y^6$

13 Write in the form $x^2 + px + q = 0$ the quadratic equation the sum
   of whose roots is 4 and the product of whose roots is $-3$.

14 If $\sin x = -\frac{1}{2} \sqrt{2}$ and $\cos x$ is negative, find $\tan x$.

15 Is the statement $x^2 - 5x = 6$ an identity; that is, is it true for all
   values of $x$?

16 Find the equation of the straight line whose slope is 2 and which
   passes through the point $(5, 6)$.

17 Find the smallest value of $x$ greater than $0^\circ$ which satisfies the
   equation $2 \cos^2 x - 3 \cos x + 1 = 0$

18 Find the value of $\cos (\sin^{-1} \frac{3}{4})$ when the angle is in the second
   quadrant.

19 Change the fraction $\frac{1}{\sqrt{1 - \cos x}}$ into an equivalent fraction having
   for its denominator $\sin x$.

20 The intensity of light on an object varies inversely as the square of
   the distance of the object from the source of light. An object is 15 feet
   from a lamp. At what distance from the lamp must the object be placed
   so that the intensity will be 9 times as great?
Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) mathematics, third year. The minimum time requirement is five recitations a week for a school year after the completion of elementary algebra.

Part II

Answer five questions from this part, selecting three questions from group I and two from group II.

Group I

Answer three questions from this group.

21 Solve the following system of equations and correctly group your answers:
\[
\frac{x}{y} - \frac{y}{x} = \frac{5}{6} \\
\frac{x}{y} \quad \frac{y}{x} = \frac{5}{6}
\]

22 Find by the use of logarithms the value of \( F \) in the formula \( F = \frac{M V^2}{R} \), when \( M = .9964 \), \( V = 34.50 \) and \( R = 19.78 \) \[10\]

23 A can do a piece of work in 10 days and B can do the same piece of work in 15 days. If A starts the work and works alone for 5 days, how long will it take A and B together to complete the work? \[7, 3\]

24 Three positive numbers, the second of which is 4, are in arithmetic progression. If 1, 5 and 21 are added to them respectively, the resulting numbers are in geometric progression. Find the original numbers. \[10\]

25 A boy throws a ball at an angle of 45° from the ground with a speed of about 57 feet a second. The path of the ball can be expressed by the equation \( y = x - \frac{x^2}{100} \), where \( x \) represents the horizontal distance in feet traveled by the ball and \( y \) the vertical distance in feet.

a) Draw a graph of this equation from \( x = 0 \) to \( x = 100 \), taking \( x \) at intervals of 10. \[8\]

b) From the graph determine the greatest height above the ground to which the ball will rise. \[1\]

c) From the graph determine the horizontal distance the ball will have traveled when it reaches the ground. \[1\]

Group II

Answer two questions from this group.

26 a) Prove the law of sines for the acute triangle. \[7\]

b) Prove the following identity:
\[
\tan A + \tan B = \frac{\sin (A + B)}{\cos A \cos B} \]

27 The sides of a triangle are 376, 423 and 587; find the largest angle correct to the nearest minute. \[10\]

28 A man in a balloon one mile high observed the angle of depression of a marker on the ground to be 27° 20'. After ascending vertically for 20 minutes at a uniform rate, he observed the angle of depression of the same marker to be 48° 10'. Find the rate of ascending in miles per hour. [Express your answer correct to the nearest tenth of a mile.] \[10\]