The University of the State of New York
254th High School Examination
MATHEMATICS — Third Year
Thursday, June 23, 1932 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.
Answer all questions in part I and five questions from part II.
Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and reduced to its simplest form.

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

In this examination the customary lettering is used. A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

In both parts of this examination the use of the slide rule will be allowed for checking; in part II all computations with tables must be shown on the answer paper.
Fill in the following lines:

Name of school........................................... Name of pupil...........................................

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each question has 2½ credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.

1. Give the product of the roots of the equation \(5x^2 + 2x - 6 = 0\)

2. Are the roots of \(4x^2 - 3x = 1\) rational or irrational?

3. Combine into a single term \(4i - \sqrt{-25}\)

4. Find the value of \(3(5)^9 + \left(\frac{1}{2}\right)^5\)

5. Write in simplest form the fourth term of \((x + \frac{1}{2})^8\)

6. Find the number whose logarithm is 9.8826—10

7. A library charges for the use of a book 25 cents for the first week and 3 cents for each additional day. Write the formula for the number of cents (c) it would cost to keep a book for n days, n being greater than 7.

8. The first term of a geometric progression is \(\frac{1}{2}\) and the fifth term is 128; find the ratio.

9. In an arithmetic progression of 5 terms, \(a = -5\) and \(l = -17\); find \(d\).

10. If \(A\) can do a piece of work in \(x\) days where \(x\) is greater than 4, after 4 days what fractional part of the work remains to be done?

11. Factor \(2\sin^2 x + 1 = \sin A - 3\)

12. Find to the nearest degree the angle made with the x-axis by the graph of the equation \(y = 2x + 3\)

13. Find \(\sin 67^\circ 41'\)

14. Find to the nearest minute the smallest positive value of \(A\) if \(\log \tan A = 0.3646\)

15. Find to the nearest foot one of the equal sides of an isosceles triangle if the third side is 809 feet and each of the equal angles is 36°.

16. If \(\tan A = \frac{4}{3}\) and \(A\) is in the third quadrant, find \(\sin A\).

17. Express in radical form the value of \(\cos 135^\circ\).

18. What is the largest positive value the sine may have?

19. Solve \(\tan^2 x = \frac{1}{3}\) for the smallest positive value of \(x\).

20. If \(A\) is an angle in the first quadrant, does cot \(A\) increase or decrease as \(\tan A\) increases?
Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in (1) elementary algebra, (2) mathematics, third year. The minimum time requirement is five recitations a week for a school year after the completion of elementary algebra.

Part II

Answer five questions from this part, selecting three questions from group I and two from group II.

Group I

Answer three questions from this group.

21 What must be the dimensions of a rectangular lot which is to contain 60 square rods and have a 38-yard fence around it? [6, 4]

22 Solve the following set of equations, group your answers and check one pair:
\[
\frac{x^2}{3} - \frac{xy}{2} = 6
\]
\[
2x + y = 4
\]
[7, 1, 2]

23 Each of two cars travels a distance of 60 miles. One travels 6 miles an hour faster than the other and makes the trip in 20 minutes less time. Find the rates of the two cars. [6, 4]

24 Given the equations \(x^2 + y^2 = 25\) and \(x - y = 2\)
   a Using the same set of axes, plot the graphs of these equations. [4, 2]
   b State how the graph shows that the solutions of this set of equations are real. [2]
   c From the graph estimate these solutions to the nearest tenth. [2]

25 The edge of one cube exceeds the edge of another cube by 2 inches. The difference between the volumes of the two cubes is 26 cubic inches. Find an edge of each cube. [4, 6]

Group II

Answer two questions from this group.

26 From the top of a hill the angles of depression of two markers one mile apart on a straight level road leading from the foot of the hill are 18° 32' and 10° 8'; how high is the hill? [10]

27 Two straight roads start from a city \(A\), making an angle of 52° with each other. One road leads to a town \(B\), which is 15 miles from \(A\), and the other leads to a town \(C\), which is 12 miles from \(A\). How long will it take a Boy Scout to walk from \(B\) to \(C\) if he walks at the rate of 3 miles an hour? [10]

28 a Solve for positive values of \(x\) less than 360°:
\[
2 \cos^2 x + 5 \sin x = 4
\]
   b Prove the following identity:
\[
(1 - \cos x) \ (\csc x + \cot x) = \sin x
\]
   [4]

29 a Solve for positive values of \(x\) less than 360°:
\[
\cos 2x + 5 \cos x = 2
\]
   b Prove the following identity:
\[
\frac{1}{2} \tan x + \tan \frac{x}{2} = \sin 2x
\]
   [4]

* These questions are based on optional topics in the syllabus.