The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Wednesday, August 13, 1969—8:30 to 11:30 a.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of \( \pi \) or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1. The measure of an interior angle of a regular polygon is 144°. Find the number of sides of the polygon.

2. The three vertices of triangle \( ABC \) lie on a circle. If \( m \angle A = 50 \) and \( m \angle B = 60 \), find the number of degrees in minor arc \( AB \).

3. In triangles \( ABC \) and \( DEF \), as shown in the accompanying figures, angle \( A \cong \) angle \( D \), angle \( B \cong \) angle \( F \), and \( AB \cong FD \).

4. If \( FE = 8 \), \( DE = 5 \), and \( AC \) is represented by \( 2x - 7 \), find the value of \( x \).

5. In the accompanying figure, \( DE \parallel AB \) and \( FF \) bisects \( \angle B \).

6. A rectangular photograph 5 inches by 9 inches is to be enlarged so that its longer dimension will be 27 inches. Find the number of inches in the shorter dimension of the enlargement.

7. Find the number of degrees in the angle that is equal in degrees to five times its complement.

8. The diagonals of a rhombus are 15 and 24. What is the area of the rhombus?

9. Find to the nearest tenth the length of the altitude to the base of an isosceles triangle one of whose legs is 20 and one of whose base angles contains 35 degrees.

10. The areas of two similar triangles are 12 and 27. If a side of the smaller triangle is 4, find the length of the corresponding side of the larger triangle.

11. Find the length of the line segment joining the points \( A \) (1,6) and \( B \) (6, -4).

12. In the accompanying figure, the radius of the smaller circle is 2 inches, and the radius of the larger circle is 3 inches.

Find in terms of \( \pi \) the number of square inches in the area of the shaded portion.

[OVER]
13 The altitude to the hypotenuse of a right triangle divides the hypotenuse into two segments. If the altitude is 4 and the shorter segment is 2, what is the length of the longer segment?

14 In a circle with radius of length 15 units, chord $AB$ is 9 units from the center. Find the number of units in $AB$.

15 The length of the median to the hypotenuse of a right triangle is represented by $2x + 3$. If the length of the hypotenuse is 18, find the value of $x$.

16 The point $M (3,2)$ is the midpoint of $AB$. If the coordinates of $A$ are $(-1,0)$, find the coordinates of $B$.

17 In the diagram, $BA$ is tangent to circle $O$ at $A$, and $BD$ is a secant intersecting the circle at $C$. If $BA = 16$ and $DB = 32$, find $BC$.

18 Two chords intersect at a point within a circle. If the lengths of the segments of one chord are $a$ and $b$ and the lengths of the segments of the other chord are $c$ and $x$, express $x$ in terms of $a$, $b$, and $c$.

19 In isosceles trapezoid $ABCD$, $AB$ and $DC$ are the bases. If $AB = 16$, $DC = 10$, and $AD = 5$, find the length of the altitude of the trapezoid.

20 Given point $P (2,3)$. If $R$ is the foot of the perpendicular from $P$ to the $x$-axis, what are the coordinates of point $R$?

23 If the distance between the centers of two circles is greater than the sum of the two radii, then the number of common tangents to the circles is

(1) 1 (3) 3
(2) 2 (4) 4

24 What is the area of an equilateral triangle whose side is 6?

(1) $9\sqrt{3}$ (3) $18\sqrt{3}$
(2) $36\sqrt{3}$ (4) 18

25 Find the area of a square whose side is $4\sqrt{2}$.

(1) $8\sqrt{2}$ (3) $16\sqrt{2}$
(2) $64$ (4) 32

26 In right triangle $ABC$, $CD$ is the altitude upon hypotenuse $AB$. The length of segment $CD$ is equal to

(1) $AD \tan A$ (3) $AC \tan B$
(2) $AD \sin A$ (4) $DB \sin B$

27 In triangle $ABC$, if the median and the altitude from vertex $C$ coincide, then triangle $ABC$ must be

(1) scalene (3) isosceles
(2) right (4) acute

28 Assume that to be in a certain mathematics class, students must have an average of 80% or more. Which statement follows from this assumption?

(1) Jean, who has an average of 85%, must be in that mathematics class.
(2) Jean, who has an average of 70%, cannot be in that mathematics class.
(3) Jean, who has an average of 80%, cannot be in that mathematics class.
(4) Jean, who is not in that mathematics class, cannot have an average of 80% or more.

29 If the measure of the vertex angle of an isosceles triangle is more than 60, then

(1) the legs of the triangle are longer than the base
(2) the measure of an exterior angle at the base is less than 120
(3) the base is the longest side of the triangle
(4) the base is equal in length to one leg

Directions (31–30): For each statement or question, write on the separate answer sheet the number preceding the word or expression that, of those given, best completes the statement or answers the question.

21 If two tangents are drawn to a circle at the ends of a chord which has an arc of measure 110, the triangle formed is

(1) acute (3) right
(2) scalene (4) obtuse

22 In a given plane, consider circles passing through point $P$ with radii of length 7. The locus of the centers of these circles is

(1) a point (3) a line
(2) two points (4) a circle

Directions (30): Leave all construction lines on the answer sheet.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove either a or b but not both: [10]
   a If two angles of a triangle are equal, the sides opposite these angles are equal.

   OR

   b The area of a regular polygon is equal to one-half the product of its perimeter and its apothem.

32 In circle O with chord RS, congruent chords SQ and RQ intersect at V. Chords RQ and SP are drawn.
   Prove: \( \triangle RSQ \cong \triangle SRP \) [10]

33 The points A (5,0), B (8,2), C (6,k), and D (3,3) are the vertices of a parallelogram.
   a Find the value of k. [4]
   b Using coordinate geometry, show that parallelogram ABCD is a square. [6]

34 a Prove that a triangle is isosceles if two of its altitudes are equal in length. [6]
   b Write the converse of the statement: a triangle is isosceles if two of its altitudes are equal in length. [2]
   c Draw and label a diagram for this converse. State in terms of the diagram what is given and what is to be proved. [2]

35 In a circle, the angle of a sector contains 48° and the area of the sector is 30π square inches. Find the number of inches.
   a in the length of the radius of the circle [5]
   b in the perimeter of the sector [Answer may be left in terms of \( \pi \)]. [5]

36 In right \( \triangle ABC \), \( CD \) is the altitude drawn to the hypotenuse \( AB \). The length of segment \( DB \) is 5 units greater than the length of segment \( AD \).
   a If \( AD = x \) and \( CD = 6 \), write an equation in terms of \( x \) which can be used to find \( AD \). [3]
   b Solve the equation written in answer to part (a) for the positive value of \( x \). [4]
   c Using the answer obtained in part (b), approximate to the nearest degree the measure of angle \( B \). [3]

37 The vertices of quadrilateral \( ABCD \) are \( A (0,0) \), \( B (a,0) \), \( C (d,c) \), and \( D (b,c) \) where \( d \neq a + b \).
   a Show that \( ABCD \) is not a parallelogram. [3]
   b Find the coordinates of point \( M \), the midpoint of \( AD \). [2]
   c Find the coordinates of point \( N \), the midpoint of \( BC \). [2]
   d Show that \( MN \parallel CD \). [3]

*This question is based on an optional topic in the syllabus.
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ANSWER SHEET

Pupil.......................................................... Teacher..........................................................

School........................................................................................................................................

Name and author of textbook used..............................................................................................

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer all questions in this part.

1. ................................................ 9. ................................................ 17. ................................................

2. ................................................ 10. ................................................ 18. ................................................

3. ................................................ 11. ................................................ 19. ................................................

4. ................................................ 12. ................................................ 20. ................................................

5. ................................................ 13. ................................................ 21. ................................................

6. ................................................ 14. ................................................ 22. ................................................

7. ................................................ 15. ................................................ 23. ................................................

8. ................................................ 16. ................................................ 24. ................................................

Questions 25 through 30 should be answered on the back of this page.

[7]
FOR TEACHERS ONLY

10

SCORING KEY

TENTH YEAR MATHEMATICS

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Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil’s work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 21-29, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 10
(2) 140
(3) 6
(4) 20
(5) $\sqrt{75}$ or $5\sqrt{3}$
(6) 15
(7) 75
(8) 180
(9) 11.5
(10) 6
(11) $\sqrt{125}$ or $5\sqrt{5}$
(12) $5\pi$
(13) 8
(14) 24
(15) 3
(16) (7,4)
(17) 8
(18) $\frac{ab}{c}$
(19) 4
(20) (2,0)
(21) 1
(22) 4
(23) 4
(24) 1
(25) 4
(26) 1
(27) 3
(28) 2
(29) 3

[OVER]
Tenth Year Mathematics — Concluded

Part II

Please refer to the Department’s pamphlet Suggestions on the Rating of Regents Examination Papers in Mathematics. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33) a 5 [4]

(34) b If a triangle is isosceles, two of its altitudes are equal. [2]

(35) a 15 [5]
   b 30 + 4\pi [5]

(36) a \frac{x}{6} = \frac{6}{x + 5} [3]
   b 4 [4]
   c 34 [3]

*(37) b \left( \frac{b}{2}, \frac{c}{2} \right) [2]
   c \left( \frac{a + d}{2}, \frac{c}{2} \right) [2]

DO YOU KNOW…

… that 400 classroom teachers were involved in preparing Regents examinations last year?

- Teachers wrote the questions.
- Other teachers assembled the examinations.
- Still other teachers reviewed the finished product.

And a committee of principals approved all of the examinations before they went to the printer.