The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, August 20, 1962 — 8:30 to 11:30 a.m., only

Name of pupil.. Name of school..

Name and author of textbook used..

Name of teacher..

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 An exterior angle at the base of an isosceles triangle is 130°. Find the number of degrees in the vertex angle. 1...............

2 Find the length of a chord of a circle of radius 13, if the chord is 5 units from the center of the circle. 2...............

3 Angle ABC inscribed in circle O intercepts arc AC. If $\angle ABC = 50°$, find the number of degrees in central angle AOC. 3...............

4 Chords AB and CD of circle O intersect inside the circle at point F. If $AF = a$, $FB = b$ and $CF = c$, express FD in terms of a, b and c. 4...............

5 In $\triangle ABC$, angle $C = 90°$ and CD is an altitude. If $CD = 6$ and $AD = 2$, find the length of DB. 5...............

6 If one diagonal of a rhombus is 6 and its area is 24, find the length of a side of the rhombus. 6...............

7 A tangent to a circle from an external point F is 10 units long. When a diameter is extended to meet the tangent at F, a secant 20 units long is formed. Find the length of the diameter. 7...............

8 The shorter side of a rectangle is 10. If a diagonal is 20, find the number of degrees in the angle which this diagonal makes with the longer side. 8...............

9 The area of an equilateral triangle is $5\sqrt{3}$. Express in radical form the length of one side. 9...............

10 Two tangents are drawn to a circle from the same external point. If the number of degrees in the angle between the two tangents is x, express in terms of x the number of degrees in the smaller of the two intercepted arcs. 10...............

11 The coordinates of the midpoint of a line segment are (3,7). If the coordinates of one end point are (—2,4), find the coordinates of the other end point. 11...............

12 If two triangles are similar and the ratio of two corresponding altitudes is 1 : 3, what is the ratio of the perimeter of the smaller to the perimeter of the larger? 12...............

13 The circumference of a circle is 24π. Find in terms of π the area of a sector whose central angle is $90°$. 13...............

14 Two sides of a parallelogram are 10 and 4 and the included angle is $45°$. Express the area in radical form. 14...............

15 The area of a trapezoid is 20, and the average of the two bases is 10. Find the altitude. 15...............

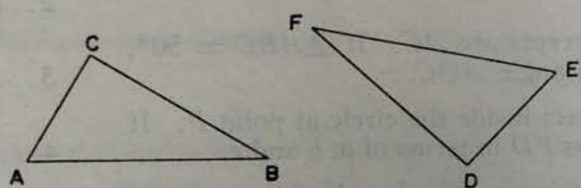
16 Find the length of the line segment joining $(3a, 5a)$ to $(0, a)$. 16...............

17 In circle O, chord AB meets diameter AOC at A. The angle between the tangent drawn at A and chord AB is $40°$. Find the number of degrees in $\angle BAC$. 17...............

18 A graph consists of the set of points such that each ordinate is ten more than twice the corresponding abscissa. Write an equation of this graph. 18...............

19 A triangle with base x and altitude y is equal in area to a parallelogram with base $2x$. Express the altitude of the parallelogram in terms of y. 19...............

20 In the triangles shown, $\angle C = \angle D$, $AC = DE$, $BC = DF$, $\angle B = (3x + 12)°$, $\angle E = (8x + 5)°$ and $\angle F = (5x - 2)°$. Find the value of x.



20...............

21 In $\triangle ABC$, line DE is drawn parallel to BC, intersecting AB in D and AC in E. If $\dfrac{AD}{DB} = \frac{1}{3}$, find the ratio of DE to BC. 21...............

Directions (22–29): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

22 In $\triangle ABC$, an exterior angle at A is $50°$. If $\angle C > 30°$, then

(1) $\angle B > 20°$
(2) $\angle B < 20°$
(3) $\angle A < 130°$
(4) $\angle A > 130°$

22.....

23 An arc of a circle is 8 units long. If the radius is also 8, the central angle subtended by this arc is

(1) less than $60°$
(2) equal to $60°$
(3) greater than $60°$ but less than $90°$
(4) greater than $90°$

23.....

24 The total number of points which are 3 units from a fixed point P and also 3 units from a line through P, is

(1) 1 (2) 2 (3) 3 (4) 4

24.....

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I do so declar...................

25 If in quadrilateral $ABCD$, diagonals AC and BD are perpendicular to each other and BD is bisected by AC, then in the quadrilateral

(1) AD must equal DC
(2) AB must equal BC
(3) AD must equal BC
(4) AD must equal AB 25......

26 A desirable property of a good definition is that it include only the descriptive characteristics that are essential. Which statement does *not* possess that property?

(1) An isosceles triangle is a triangle two of whose sides are equal.
(2) A rhombus is a parallelogram with two adjacent sides equal.
(3) A rectangle is a parallelogram with four right angles.
(4) A regular polygon is a polygon which is both equilateral and equiangular. 26......

27 Two equal circles with centers at P and Q do not intersect. A line parallel to PQ meets circle P in A and B and circle Q in C and D. Then

(1) $AB = CD$
(2) $AB = PQ$
(3) $AB > CD$
(4) $AB > PQ$ 27......

28 The contrapositive of a theorem is the inverse of its converse. Consider the theorem " If two sides of a triangle are equal, the angles opposite those sides are equal." Which statement is the contrapositive of the above theorem?

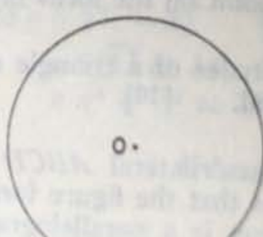
(1) If two angles of a triangle are unequal, the sides opposite those angles are unequal.
(2) If two sides of a triangle are unequal, the angles opposite those sides are unequal.
(3) If two angles of a triangle are equal, the sides opposite those angles are equal.
(4) If a triangle is isosceles, its base angles are equal. 28......

29 Three statements — a, b, c — are listed below. In which order should they be rearranged to present a logical sequence?

a The sum of the angles of a triangle is 180°.
b If two triangles agree in two angles, then the third angles are equal.
c Through a point, not on a line, one and only one straight line may be constructed parallel to the given line.

(1) $a\ c\ b$ (2) $c\ a\ b$ (3) $b\ c\ a$ (4) $c\ b\ a$ 29......

30 Inscribe a square $ABCD$ in circle O. [Leave all construction lines on the paper.]



Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* *a* or *b*:

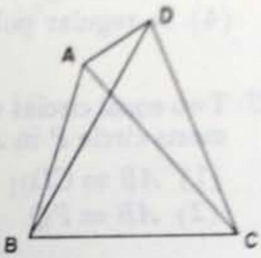
a An angle inscribed in a circle is measured by one-half its intercepted arc. [Consider only the case where one side of the angle is a diameter.] [10]

OR

b If, in a right triangle the altitude is drawn upon the hypotenuse,

(1) the two triangles thus formed are similar to the given triangle and similar to each other [7] and

(2) each leg of the given triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse [3]



32 Given: quadrilateral $ABCD$ as shown in the figure with $BD = DC$.
Prove: $AC > AB$. [10]

33 In triangle ABC, medians AD and BE intersect at O, and line ED is drawn.

a Prove: $EO:OB = 1:2$ [6]

b Using K to represent the area of $\triangle EOD$, express in terms of K the areas of $\triangle AOB$ and $\triangle DOB$. [4]

34 A regular pentagon is inscribed in a circle of radius 7.

a Find to the *nearest tenth* the length of a side of the pentagon. [3]

b Find to the *nearest tenth* the length of the apothem of the pentagon. [2]

c Using the results obtained in parts *a* and *b*, find to the *nearest integer* the area of the pentagon. [3]

d Using the approximation $\pi = \frac{22}{7}$, find to the *nearest integer* the area which is within the circle but *not* in the pentagon. [2]

35 *a* On coordinate paper and using the same set of axes, draw

(1) the locus of points whose abscissa is 3 [2]

(2) the locus of points whose ordinate is —2 [2]

b The two loci meet at point A. What are the coordinates of A? [1]

c Find the distance from A to the origin. [2]

d Using the same set of axes as in part *a*, draw the locus of points whose distance from the origin is equal to OA. [2]

e If (x,y) is any point on the locus drawn in part *d*, write an equation of that locus. [1]

36 Prove: If two altitudes of a triangle are equal, and one of these altitudes is also a median, the triangle is equilateral. [10]

*37 The vertices of quadrilateral $ABCD$ are $A(0,4)$, $B(4,2)$, $C(6,—4)$ and $D(—1,—2)$. Show by means of slopes that the figure formed by joining the midpoints of the sides of the quadrilateral in succession is a parallelogram. [10]

* This question is based on an optional topic in the syllabus.

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FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING TENTH YEAR MATHEMATICS

Monday, August 20, 1962 — 8:30 to 11:30 a.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–29, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 80
(2) 24
(3) 100
(4) $\frac{ab}{c}$
(5) 18
(6) 5
(7) 15
(8) 30
(9) $2\sqrt{5}$
(10) $180 - x$
(11) (8, 10)
(12) 1:3
(13) 36π
(14) $20\sqrt{2}$
(15) 2
(16) $5a$
(17) 50
(18) $y = 2x + 10$
(19) $\frac{1}{4}y$
(20) 7
(21) $\frac{1}{4}$
(22) 2
(23) 1
(24) 2
(25) 4
(26) 3
(27) 1
(28) 1
(29) 2

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33) *b* $4K, 2K$ [2, 2]

(34) *a* 8.2 [3]
b 5.7 [2]
c 117 [3]
d 37 [2]

(35) *b* (3, —2) [1]
c $\sqrt{13}$ *or* 3.6 [2]
e $x^2 + y^2 = 13$ [1]

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