TENTH YEAR MATHEMATICS

Monday, June 21, 1982 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

On page 9 you will find the "Tables of Natural Trigonometric Functions" which you may need to answer some questions in this examination. Fold this page along the perforations, and tear it off also slowly and carefully.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of \( \pi \) or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1. If each exterior angle of a regular polygon measures 90°, how many sides does the polygon have?

2. A person 6 feet tall casts a shadow 4 feet long. At the same time, a nearby tower casts a shadow 32 feet long. How many feet are in the height of the tower?

3. In triangle \( ABC \), \( \angle A = 85 \) and \( \angle C = 67 \). Which side of triangle \( ABC \) is the shortest side?

4. The lengths of the diagonals of a rhombus are 10 and 12. What is the area of the rhombus?

5. Find the radius of a circle whose area is 64\( \pi \).

6. In the accompanying diagram of circle \( O \), diameter \( AB \), chord \( BC \), and radius \( OC \) are drawn. If \( \angle AOC = 76 \), find \( \angle ABC \).

7. The length of one side of a rectangle is 3 and the length of a diagonal is 5. Find the area of the rectangle.

8. In the accompanying diagram of parallelogram \( ABCD \), diagonals \( AC \) and \( BD \) intersect at \( E \). If \( EC = 31 \), \( EB = 27 \), and \( AE = 4x - 5 \), find the value of \( x \).

9. What is the area of the triangle whose vertices are \( A(4, 0) \), \( B(0, 0) \), and \( C(0, 4) \)?

10. In the accompanying diagram of circle \( O \) with diameter \( AB \), chords \( AC \) and \( BC \) are drawn. If \( AB = 13 \) and \( AC = 5 \), find the length of \( BC \).

11. In a trapezoid, the lengths of the bases are 2 and 3 and the altitude is 4. Find the area of the trapezoid.

12. In the accompanying diagram, \( \overrightarrow{AB} \parallel \overrightarrow{CD} \), \( \overrightarrow{AE} \perp \overrightarrow{CB} \), \( \angle BCD = 2x \), and \( \angle BAE = 3x \). Find the value of \( x \).

Directions (13–29): Write in the space provided on the separate answer sheet the numeral preceding the expression that best completes each statement or answers each question.

13. As shown in the diagram at the right, three straight lines intersect at one point. If \( \angle 2 \equiv \angle 4 \), then \( \angle 1 + \angle 3 + \angle 4 \) is equal to
   (1) 180
   (2) 200
   (3) 222
   (4) 250

14. The coordinates of the vertices of rectangle \( ABCD \) are \( A(1, 4) \), \( B(1, 1) \), \( C(7, 1) \), and \( D(7, 4) \). The coordinates of the point where the diagonals of the rectangle intersect are
   (1) \( (1, \frac{5}{2}) \)
   (2) \( (7, \frac{5}{2}) \)
   (3) \( (8, 5) \)
   (4) \( (4, \frac{5}{2}) \)
15 In parallelogram $ABCD$, $m \angle A = 50$. What is $m \angle C$?
   (1) 130  
   (2) 50  
   (3) 40  
   (4) 25

16 If $\overrightarrow{AB} \parallel \overrightarrow{CD}$ and the slope of $\overrightarrow{AB}$ is $\frac{1}{2}$, then the slope of $\overrightarrow{CD}$ is
   (1) $-2$  
   (2) $2$  
   (3) $-\frac{1}{2}$  
   (4) $\frac{1}{2}$

17 Given the true statement, "If it is raining, then there are clouds in the sky." Which is also a true statement?
   (1) If there are clouds in the sky, then it is raining.
   (2) If it is not raining, then there are no clouds in the sky.
   (3) If there are no clouds in the sky, then it is not raining.
   (4) If there are no clouds in the sky, then it is raining.

18 An equation of the locus of points whose abscissas are equal to their ordinates is
   (1) $y = x$  
   (2) $x = 0$  
   (3) $y = 0$  
   (4) $x + y = 0$

19 In the accompanying diagram, $\triangle ABC$ is similar to $\triangle DEF$, $AC = 3$, and $AB = BC = 6$. If $DF = 4$, what is the perimeter of $\triangle DEF$?

20 In right triangle $DEF$, $m \angle F = 90$. Which ratio represents $\sin E$?
   (1) $\frac{DF}{DE}$  
   (2) $\frac{DE}{DF}$  
   (3) $\frac{EF}{DE}$  
   (4) $\frac{DF}{EF}$

21 Congruent polygons are best defined as polygons in which
   (1) all the corresponding sides are in proportion
   (2) all the corresponding angles are congruent
   (3) all the corresponding angles are congruent and all the corresponding sides are congruent
   (4) all the corresponding angles are congruent and all the corresponding sides are in proportion

22 Two chords intersect inside a given circle. The lengths of the segments of the first chord are 6 and 15, and the length of the second chord is 23. The lengths of the segments of the second chord are
   (1) 3 and 20  
   (2) 5 and 18  
   (3) $11 \frac{1}{2}$ and $11 \frac{1}{2}$  
   (4) 10 and 13

23 In triangle $ABC$, $m \angle A = 30$ and $m \angle C = 90$. If $BC = 4$, then $CA$ equals
   (1) 8  
   (2) 2  
   (3) $4\sqrt{3}$  
   (4) 4

24 What is the slope of the straight line which passes through the points (2,5) and (1,6)?
   (1) 1  
   (2) 2  
   (3) 0  
   (4) $-1$

25 Given $\overrightarrow{PA}$ and $\overrightarrow{PB}$ tangent to circle $O$ at points $A$ and $B$, respectively, and radii $OA$ and $OB$ are drawn. Which is always true?
   (1) $m \angle P = m \angle AOB$  
   (2) $m \angle P > m \angle AOB$  
   (3) $m \angle P + m \angle AOB = 90$  
   (4) $m \angle P + m \angle AOB = 180$

26 Two roads intersect at right angles. A pole is 30 meters from one road and also 40 meters from the other road. How far is the pole from the point where the roads intersect?
   (1) 30 m  
   (2) 40 m  
   (3) 50 m  
   (4) 60 m

27 In the accompanying figure, the length of the radius of circle $O$ is 6. What is the area of sector $AOB$ whose central angle measures $90^\circ$?

\[ \text{(1) } 9\pi \quad \text{(2) } \frac{3\pi}{2} \quad \text{(3) } 3\pi \quad \text{(4) } \frac{\pi}{4} \]
28 The corresponding sides of two similar triangles are in the ratio 2:3. What is the ratio of the area of the smaller triangle to the area of the larger triangle?
(1) 1:√6
(2) 2:3
(3) √2:√3
(4) 4:9

Directions (30): Leave all construction lines on the answer sheet.

30 On the answer sheet, construct the bisector of ∠ABC.

29 Given the statement: "A triangle cannot have two right angles." In order to prove this statement by the indirect method, it should be assumed that a triangle:
(1) does not have a right angle
(2) has two right angles
(3) has one right angle
(4) does not have two right angles
Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove either a or b but not both.
   a The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs. OR
   b The area of a regular polygon is equal to one-half the product of its perimeter and the length of its apothem. [10]

32 Given: circle O with chord \( \overline{AB} \) parallel to chord \( \overline{CD} \), chords \( \overline{AD} \) and \( \overline{BC} \) intersect at E.

![Diagram showing circle O with chords AD and BC intersecting at E.]

Prove: \( \overline{AD} \cong \overline{BC} \) [10]

33 A regular polygon with 9 sides is inscribed in a circle of radius 12.
   a Find the length of an apothem of the polygon to the nearest tenth. [4]
   b Find the length of a side of the polygon to the nearest integer. [4]
   c Using the results obtained in parts a and b, find the area of the polygon to the nearest integer. [8]

34 Given: circle O, chord \( \overline{BGF} \) is parallel to secant \( \overline{CDE} \), B is the midpoint of \( \overline{AD} \), secant \( \overline{CBA} \), and chords \( \overline{AF} \) and \( \overline{AGD} \).

![Diagram showing circle O with chords AF and AGD.]

Prove: \( \frac{CD}{BA} = \frac{CA}{BF} \) [10]

35 Given quadrilateral \( ABCD \) with coordinates A(2,4), B(−5,2), C(−2,−1), and D(5,1). Show by means of coordinate geometry that quadrilateral \( ABCD \) is a parallelogram and state a reason for your conclusion. [10]

36 Given: isosceles triangle \( ABC \) with \( \overline{CA} = \overline{CB} \), \( \overline{CFB} \), and \( \overline{BSA} \).

Prove: \( AF > FS \) [10]

*37 The coordinates of the vertices of trapezoid \( ABCD \) are \( A(0,−2) \), \( B(8,6) \), \( C(2,8) \), and \( D(−2,4) \).
   a Write an equation of the line through A and B. [3]
   b If the perpendicular from D to \( \overline{AB} \) intersects \( \overline{AB} \) at E, write an equation of the line through D and E. [4]
   c Using the results from parts a and b, find the coordinates of E. [3]

*This question is based on an optional topic in the syllabus.
# Tables of Natural Trigonometric Functions

(For use with 9th and 10th Year Mathematics Regents Examinations)

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Math. 10-June '82
The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION
TENTH YEAR MATHEMATICS
Monday, June 21, 1982 — 1:15 to 4:15 p.m., only

ANSWER SHEET

Pupil............................................................................. Teacher..........................................................................

School........................................................................................................................................

Name and author of textbook used................................................................................................

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer all questions in this part.

1. ........................................ 11. .............................. 21. ..............................
2. ........................................ 12. .............................. 22. ..............................
3. ........................................ 13. .............................. 23. ..............................
4. ........................................ 14. .............................. 24. ..............................
5. ........................................ 15. .............................. 25. ..............................
6. ........................................ 16. .............................. 26. ..............................
7. ........................................ 17. .............................. 27. ..............................
8. ........................................ 18. .............................. 28. ..............................
9. ........................................ 19. .............................. 29. ..............................
10. ....................................... 20. ............................ 30 Answer question 30 on the other side of this sheet.

Math. 10—June '82

[11]

[OVER]
Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

______________________________
Signature

Math. 10-June '82
FOR TEACHERS ONLY

SCORING KEY

TENTH YEAR MATHEMATICS

Monday, June 21, 1982 — 1:15 to 4:15 p.m., only

Use only red ink or red pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 13–29, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 4
(2) 48
(3) $AC$ or $AC$ or $b$
(4) 60
(5) 8
(6) 38
(7) 12
(8) 9
(9) 8
(10) 12
(11) 10
(12) 18
(13) 1
(14) 4
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(24) 4
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(26) 3
(27) 1
(28) 4
(29) 2
(30) construction
Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33) \( a \ 11.3 \quad [4] \)

\( b \ 8 \quad [4] \)

\( c \ 407 \quad [8] \)

(37) \( a \ y = x - 2 \)

\( \quad \text{or} \)

\( y - 6 = 1(x - 8) \quad [3] \)

\( b \ y = -x + 2 \)

\( \quad \text{or} \)

\( y - 4 = -1(x + 2) \quad [4] \)

\( c \ (2,0) \text{ or } x = 2 \)

\( y = 0 \quad [3] \)