The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Friday, June 18, 1971 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. On page 5, which is perforated, you will find the “Tables of Natural Trigonometric Functions,” which you will need to answer some questions in this examination. Fold this page along the perforation, and tear it off. When you have torn off these two pages and finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of \( \pi \) or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1. If \( \angle AEC = (5x + 20) \) and \( \angle DEB = (3x + 50) \), find \( x \).

2. In triangle \( ABC \), the measure of angle \( C \) is twice the measure of angle \( A \). The measure of an exterior angle at \( B \) is 150°. Find \( \angle A \).

3. What is the area of an equilateral triangle which has a side of length 10?

4. Express in terms of \( \pi \) the area of a sector whose central angle measures 60° in a circle whose radius is 6.

5. In triangle \( ABC \), \( \angle B > \angle C \) and \( \angle C > \angle A \). Which side of the triangle has the greatest length?

6. The area of a trapezoid is 44. If the length of the altitude is 8 and the length of one base is 3, find the length of the other base.

7. The length of the hypotenuse of a right triangle is 15 and the length of one leg is 12. Find the length of the other leg.

8. The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments whose lengths are 8 and 2. What is the length of the altitude?

9. The areas of two squares are, respectively, 16 and 100. What is the ratio of a side of the smaller square to a side of the larger square?

10. The lengths of the sides of a triangle are 6, 7, and 8. Find the perimeter of a similar triangle whose longest side is 16.

11. In the accompanying diagram, quadrilateral \( ABCD \) is inscribed in circle \( O \).

If \( \overline{AB} = 119 \), and \( \overline{BC} = 73 \), find \( \angle ADC \).

12. In convex quadrilateral \( ABCD \),
\( m \angle B + m \angle D = 180 \).
The measure of angle \( A \) is 50° more than the measure of angle \( C \). Find \( m \angle C \).

13. In the figure below, \( \overline{AD}, \overline{AB}, \) and \( \overline{BE} \) are tangent to circle \( O \) at points \( D, F, \) and \( E \), respectively.

If \( AD = 2 \) and \( EB = 4 \), find \( AB \).
14 The coordinates of $A$ and $B$ are $(2a,2b)$ and $(4a,6b)$, respectively. Express in terms of $a$ and $b$ the coordinates of the midpoint of $AB$.

15 The vertices of rectangle $ABCD$ are the points $A(0,0)$, $B(8,0)$, $C(8,k)$, and $D(0,5)$. Find $k$.

16 Two parallel lines are cut by a transversal. One of the two interior angles on the same side of the transversal measures five times the other. Find the number of degrees in the measure of the smaller angle.

17 Find the number of degrees in the measure of each exterior angle of a regular polygon of 10 sides.

Directions (18–29): For each statement or question, write on the separate answer sheet the number preceding the word or expression that, of those given, best completes the statement or answers the question.

18 A median drawn to a side of a triangle divides the triangle into two triangles which must be

(1) equal in area   (3) similar
(2) congruent   (4) equal in perimeter

19 In the diagram below, $m \angle C = 90^\circ$, $m \angle A = 32^\circ$, and $AC = 8$.

An equation that can be used to find $AB$ is

(1) $\tan 32^\circ = \frac{AB}{8}$  (3) $\cos 32^\circ = \frac{8}{AB}$
(2) $\tan 32^\circ = \frac{8}{AB}$  (4) $\cos 32^\circ = \frac{AB}{8}$

20 Distinct points which lie on the same line are said to be

(1) coincident   (3) collinear
(2) intersecting   (4) vertical

21 The lengths of two sides of a triangle are 6 and 13, respectively. The length of the third side may be

(1) 7   (3) 19
(2) 11   (4) 20

22 If the intersection of the perpendicular bisectors of the sides of a triangle lies on one of the sides of the triangle, the triangle must be

(1) acute   (3) scalene
(2) obtuse   (4) right

23 Which arrangement of the following geometric sets is an order such that each set after the first is a subset of the preceding sets?

(1) polygons, quadrilaterals, trapezoids, isosceles trapezoids
(2) quadrilaterals, polygons, trapezoids, isosceles trapezoids
(3) polygons, trapezoids, quadrilaterals, isosceles trapezoids
(4) polygons, quadrilaterals, isosceles trapezoids, trapezoids

24 The segments of one of two chords intersecting within a circle are $a$ and $b$ and the segments of the other are $c$ and $d$. Which equation expresses the relationship among $a$, $b$, $c$, and $d$?

(1) $a + b = c + d$  (3) $\frac{a}{b} = \frac{c}{d}$
(2) $ab = cd$  (4) $\frac{a}{d} = \frac{b}{c}$

25 Which statement is true?

(1) All parallelograms are similar.
(2) All isosceles trapezoids are similar.
(3) All regular polygons are similar.
(4) All squares are similar.

26 A tangent and a secant are drawn to a circle from an external point. Their lengths are represented by $p$ and $q$, respectively. The length of the external segment of the secant in terms of $p$ and $q$ is

(1) $\frac{p^2}{q}$  (3) $pq$
(2) $\sqrt{pq}$  (4) $p + q$

27 If two circles are externally tangent, what is the greatest number of common tangents which may be drawn to them?

(1) 1   (3) 3
(2) 2   (4) 4

28 Points $A$ and $B$ are 4 inches apart. The exact number of points equidistant from both $A$ and $B$ and also 3 inches from $A$ is

(1) 1   (3) 3
(2) 2   (4) 0

29 In which quadrilateral are the diagonals always congruent?

(1) rhombus   (3) parallelogram
(2) trapezoid   (4) rectangle

Directions (30): Leave all construction lines on the answer sheet.

30 On the answer sheet, locate by construction the point $P$ on side $DC$ of parallelogram $ABCD$ which is equidistant from the points $A$ and $B$. 

Math 10–June '71
Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove either $a$ or $b$ but not both. [10]

a The sum of the angles of a triangle is equal to a straight angle.

OR

b An angle formed by two secants, drawn to a circle from a point outside the circle, is measured by one-half the difference of the intercepted arcs.

32 Given: \( \overrightarrow{DB} \) is tangent to circle \( O \) at \( B \), \( \overline{AB} \) is a diameter of circle \( O \), and chord \( \overline{AC} \) extended intersects \( \overrightarrow{BD} \) at \( D \)

\[
\text{Prove: } \triangle AOB \sim \triangle BCD \quad [8]
\]

\[
\frac{\overline{AB}}{\overline{BD}} = \frac{\overline{AC}}{\overline{BC}} \quad [2]
\]

33 Using the accompanying figure and coordinates, express in terms of \( a \) and \( b \):

a the coordinates of \( M \), the midpoint of \( \overline{AB} \) \quad [2]

b the length of \( \overline{BM} \) \quad [2]

c the length of \( \overline{CM} \) \quad [2]

d the area of triangle \( \triangle ABC \) \quad [2]

e the area of triangle \( \triangle BMC \) \quad [2]

34 Given: circle \( O \), inscribed \( \triangle SAB \), \( \overline{SA} \equiv \overline{SB} \), and diameter \( \overline{ST} \) intersecting \( \overline{AB} \) at \( R \)

\[
\text{Prove: } \overline{ST} \perp \overline{AB} \quad [10]
\]

35 Given: circle \( O \) with secant \( \overline{CBA} \) and \( \overline{CD} \) tangent to circle \( O \) at \( D \); chord \( \overline{EG} \) intersecting chord \( \overline{DB} \) at \( F \); \( \overrightarrow{EC} \parallel \overrightarrow{DC} \); \( \overrightarrow{EG} \) meeting \( \overline{AC} \) at \( H \);

\( m\angle E = 50^\circ \), \( m\angle E = 140^\circ \), and \( m\angle B = 80^\circ \)

\[
\text{Find: } a \ m\angle 1 \quad [2] \\
b \ m\angle 2 \quad [3] \\
c \ m\angle 3 \quad [2] \\
d \ m\angle 4 \quad [3]
\]

36 Prove: Any segment joining the vertex of an isosceles triangle to a point of the base, other than its endpoints, is shorter than a leg of the triangle. \quad [10]

*37 The coordinates of the vertices of triangle \( \triangle ABC \) are \( A \ (2,3) \), \( B \ (9,-2) \), and \( C \ (5,8) \). Median \( \overline{AE} \) is drawn to side \( \overline{BC} \).

a Find the slope of \( \overrightarrow{AC} \). \quad [2]

b Write an equation of the straight line which has \( y \)-intercept 3 and is parallel to \( \overrightarrow{AC} \). \quad [3]

c Find the slope of \( \overrightarrow{AE} \). \quad [3]

d Find the area of triangle \( \triangle ACE \). \quad [2]

* This question is based on an optional topic in the syllabus.
### Tables of Natural Trigonometric Functions

(For use with 9th and 10th Year Mathematics Regents Examinations)

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The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Friday, June 18, 1971—1:15 to 4:15 p.m., only

ANSWER SHEET

Pupil........................................................................ Teacher.........................................................

School.................................................................................................................................

Name and author of textbook used...........................................................................................

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer all questions in this part.

1................................................................. 11................................................................. 21.................................................................

2................................................................. 12................................................................. 22.................................................................

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Answer question 30 on the back of this page.
FOR TEACHERS ONLY

SCORING KEY

TENTH YEAR MATHEMATICS

Friday, June 18, 1971 — 1:15 to 4:15 p.m., only

Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 18–29, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 15  (11) 96  (21) 2
(2) 50  (12) 65  (22) 4
(3) $25\sqrt{3}$  (13) 6  (23) 1
(4) $6\pi$  (14) $(3a, 4b)$  (24) 2
(5) $\overline{AC}$  (15) 5  (25) 4
(6) 8  (16) 30  (26) 1
(7) 9  (17) 36  (27) 3
(8) 4  (18) 1  (28) 2
(9) $\%$  (19) 3  (29) 4
(10) 42  (20) 3

[OVER]
Part II

Please refer to the Department's pamphlet Suggestions on the Rating of Regents Examination Papers in Mathematics. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33) a \((a,b)\) [2]
-b \(\sqrt{a^2 + b^2}\) [2]
-c \(\sqrt{a^2 + b^2}\) [2]
d \(2ab\) [2]
e \(ab\) [2]

(37) a \(\frac{9}{10}\) [2]
b \(y = \frac{9}{10}x + 3\) [3]
c 0 [3]
d 12.5 [2]

(35) a 95 [2]
b 45 [3]
c 50 [2]
d 45 [3]

DO YOU KNOW . . .

. . . that classroom teachers returned over 3,700 Regents examination evaluation forms to the Education Department last year? The comments and suggestions made by these teachers were carefully reviewed by the Department subject-matter and testing specialists and by the teachers who prepared this year's examinations.

Be sure to fill out the Evaluation Form and give it to your principal for return in the Regents box. Your comments about the Regents examinations are important! They will be taken into consideration when future examinations are prepared.