The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, June 16, 1969—1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of \( \pi \) or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1. How many degrees are there in the sum of the interior angles of a convex polygon which has 8 sides?

2. In circle \( O \), central angle \( AOB \) measures 100°. If \( P \) is any point on minor arc \( AB \), find the number of degrees in the measure of angle \( APB \).

3. Corresponding sides of two similar polygons are in the ratio of 2:5. If the perimeter of the smaller polygon is 100, find the perimeter of the larger polygon.

4. Chord \( CD \) is 3 inches from the center of circle \( O \). If \( CD \) is 8 inches, find the number of inches in the length of a radius of the circle.

5. In the accompanying figure, \( \vec{AB} \) is parallel to \( \vec{CD} \) and \( \vec{EG} \) bisects \( \angle BEF \). If \( m \angle FGE = 90 \) and \( m \angle FEG = 25 \), find \( m \angle GFD \).

6. In circle \( O \), diameter \( AB \) is one side of an inscribed isosceles triangle. If the length of the radius of the circle is 6 inches, what is the number of inches in the length of one of the equal sides of the triangle?

7. If the coordinates of \( A \) and \( B \) are \((6,1)\) and \((-2,7)\), respectively, find the coordinates of the midpoint of \( AB \).

8. Find the area of a rectangle with base of length 12 and diagonal of length 13.

9. Find the number of inches in the circumference of a circle in which the longest chord that can be drawn measures 16 inches.

10. In the accompanying figure, the large rectangle has been divided into a square and three smaller rectangles. If the areas of the square and two of the rectangles are \( k^2 \), \( 4k \), and \( 8k \), respectively, what is the numerical value of the area of the shaded rectangle?

11. Write an equation of the locus of points such that twice their abscissas exceeds their ordinates by 5.

12. From external point \( B \), \( AB \) is drawn tangent to circle \( O \) at \( A \). Secant \( BCD \) intersects the circle at \( C \) and \( D \). If \( CB = 4 \) and \( CD = 12 \), find \( AB \).

13. If \( \cos x = \sin 75^\circ \), how many degrees are there in the measure of angle \( x \)?

14. The radius of a circle is 9 and the angle of a sector of the circle is \( 40^\circ \). Find the area of the sector.

15. Given: \( AB \) and \( CD \) intersect at point \( E \) so that \( AC \) is parallel to \( DB \). If \( AC = 3 \), \( DB = 4 \), and \( AB = 14 \), find \( AE \).

16. The length of a side of an equilateral triangle is \( 8\sqrt{3} \). Find the length of an altitude of this triangle.

17. From a point on level ground 10 meters from the foot of a vertical telephone pole, the angle of elevation of the top of the pole measures \( 32^\circ \). Approximate the height of the telephone pole to the nearest meter.
Directions (18–28): For each statement or question, write on the separate answer sheet the number preceding the word or expression that, of those given, best completes the statement or answers the question.

18 The supplement of the complement of an acute angle is always
(1) an acute angle  (2) an obtuse angle  (3) a straight angle  (4) a right angle

19 Two isosceles triangles with equal vertex angles are always
(1) congruent  (2) equilateral  (3) right  (4) similar

20 In the accompanying figure, $\overline{AB} \cong \overline{AC}$. It can be proved that $\overline{CD} \cong \overline{BE}$ if it is also known that

(1) $\angle 1 \cong \angle 2$  (2) $\angle 3 \cong \angle 4$  (3) $\angle 3 \cong \angle 5$  (4) $\angle 4 \cong \angle 6$

21 What is the length of $\overline{AB}$ if point $A$ has the coordinates $(2,-1)$ and point $B$ has the coordinates $(10,3)$?

(1) $4\sqrt{3}$  (2) $4\sqrt{5}$  (3) $2\sqrt{17}$  (4) $2\sqrt{37}$

22 The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments whose lengths are 4 and 5, respectively. The length of the shorter leg of the right triangle is

(1) $2\sqrt{5}$  (2) $3\sqrt{5}$  (3) 6  (4) 8

24 What is the area of a circle circumscribed about a square whose apothem has a length equal to 1?

(1) $\pi$  (2) $2\pi$  (3) $2\sqrt{2}\pi$  (4) $4\pi$

25 Triangle $ABC$ is an equilateral triangle with $AB = s$ inches. The number of points 1 inch from vertex $A$ and 4 inches from side $BC$ is

(1) 1  (2) 2  (3) 3  (4) 0

26 Two secants intersecting outside a circle intercept arcs whose measures in degrees are $7x$ and $9x$. The measure in degrees of the angle formed by the two secants is represented by

(1) $x$  (2) $2x$  (3) $8x$  (4) $16x$

27 In considering the lengths of two line segments, $AB$ and $CD$, a student argues that either $AB$ is greater than $CD$, $AB$ is less than $CD$, or $AB$ is equal to $CD$. He proceeds to establish that $AB$ is not equal to $CD$ and that $AB$ is not greater than $CD$ and concludes that $AB$ is less than $CD$. The student used which method of reasoning?

(1) reasoning from an inverse  (2) direct reasoning  (3) circular reasoning  (4) indirect reasoning

28 In the accompanying diagram, $\overline{QR} \cong \overline{QS}$ and the points $R$, $S$, and $P$ are collinear (with $S$ between $P$ and $R$). It is always true that

(1) $\angle 1 > \angle 2$  (2) $\angle 1 > \angle 4$  (3) $\angle 3 > \angle 4$  (4) $\angle 4 > \angle 2$

Directions (29–30): Leave all construction lines on the answer sheet.

29 On the answer sheet, by construction inscribe a square in circle $O$.

30 Given an angle $ABC$ of measure $p$. On the answer sheet, construct and label an angle $ABD$ whose measure is $\frac{p}{4}$.
Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove either a or b but not both: [10]
   a An angle formed by two chords intersecting inside a circle is measured by one-half the sum of the intercepted arcs.

   OR

   b The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

32 Given: \( MPQN \) with \( MP = OQ \) and \( \angle KPO \equiv \angle KOP \) as shown in the accompanying figure.

Prove: \( \triangle KMN \) is an isosceles triangle. [10]

33 Write the letters a through e on your answer paper and next to each letter write the number of the graph, chosen from the diagram below, which best corresponds to the locus description given in a through e. [10]

\[ a \text{ The locus of points 3 units from the y-axis} \]
\[ b \text{ The locus of points 3 units from the origin} \]
\[ c \text{ The locus of points whose abscissas are 3 times their ordinates} \]
\[ d \text{ The locus of points equidistant from the coordinate axes} \]
\[ e \text{ The locus of points equidistant from the points (3,0) and (3,6)} \]
34 Given: $\overline{EAB}$ and parallelogram $ABCD$ as shown in the accompanying figure.

Prove: $\triangle ABC \cong \triangle CDE$ \hspace{1cm} [10]

35 In the figure, the center of circle $O$ is at the origin.

From external point $A (r,10)$, tangent lines $\overrightarrow{AB}$ and $\overrightarrow{AP}$ are drawn to the circle with points of contact at $P(5,0)$ and $B(-3,s)$. \hspace{1cm} [10]

$\overrightarrow{AP}$

$A$

$B$

$O$

$P$

$x$

$y$

$a$ Find the value of $r$.

$b$ Write an equation of tangent line $\overrightarrow{AP}$.

c Write an equation of circle $O$.

d Find the value of $s$.

e Find $AB$.

36 Given: $\angle 1 \cong \angle 2$

$\overline{CD} \parallel \overline{AB}$

Prove: $a \overrightarrow{BC} \cong \overrightarrow{CD}$ \hspace{1cm} [3]

$b \overline{AB} = \overline{AE}$ \hspace{1cm} [6]

$c \overline{CD} = \overline{EC}$ \hspace{1cm} [1]

*37 In isosceles triangle $ABC$ with vertices $A (3,-1)$, $B (7,3)$, and $C (-1,7)$, $\overline{CD}$ is the altitude to $\overline{AB}$.

$a$ If the slope of $\overline{CD}$ is $-1$, write an equation of the line passing through $C$ and $D$. \hspace{1cm} [2]

$b$ Write an equation of the line passing through $A$ and $B$. \hspace{1cm} [2]

$c$ Using coordinate geometry, show that the altitude of isosceles $\triangle ABC$ intersects the base $\overline{AB}$ at its midpoint. \hspace{1cm} [6]

* This question is based on an optional topic in the syllabus.
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ANSWER SHEET

Pupil. 
Teacher. 

School. 

Name and author of textbook used. 

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer all questions in this part.

1. 9. 17. 
2. 10. 18. 
3. 11. 19. 
4. 12. 20. 
5. 13. 21. 
7. 15. 23. 
8. 16. 24. 

Questions 25 through 30 should be answered on the back of this page.
FOR TEACHERS ONLY

SCORING KEY

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Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 18—28, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 1080
(2) 130
(3) 250
(4) 5
(5) 65
(6) $6\sqrt{2}$ or $\sqrt{72}$
(7) (2, 4)
(8) 60
(9) $16\pi$
(10) 32
(11) $y = 2x - 5$
(12) 8
(13) 15
(14) $9\pi$
(15) 6
(16) 12
(17) 6
(18) 2
(19) 4
(20) 2
(21) 2
(22) 3
(23) 3
(24) 2
(25) 4
(26) 1
(27) 4
(28) 2
Tenth Year Mathematics — concluded

Part II

Please refer to the Department’s pamphlet Suggestions on the Rating of Regents Examination Papers in Mathematics. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

33 a 7 [2]  
  b 4 [2]  
  c 1 [2]  
  d 2 [2]  
  e 6 [2]  

*37 a \( y = -x + 6 \) or \( x + y = 6 \) [2]  
  b \( y = x - 4 \) or \( x - y = 4 \) [2]

35 a 5 [2]  
  b \( x = 5 \) [2]  
  c \( x^2 + y^2 = 25 \) [2]  
  d 4 [2]  
  e 10 [2]

DO YOU KNOW...

. . . that 400 classroom teachers were involved in preparing Regents examinations last year?

• Teachers wrote the questions.
• Other teachers assembled the examinations.
• Still other teachers reviewed the finished product.

And a committee of principals approved all of the examinations before they went to the printer.