Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of $\pi$ or in radical form.

1. Find a diagonal of a square whose side is 6.
2. A diagonal of a rectangle is 17 and the length is 15. Find the width of the rectangle.
3. In parallelogram $ABCD$, $AB = 10$, $AD = 6$ and angle $A = 30^\circ$. Find the area of the parallelogram.
4. The area of a rhombus is 60 and one of its diagonals is 12. Find the other diagonal.
5. A secant and a tangent are drawn to a circle from an external point. The secant is 12 and its external segment is 3. Find the length of the tangent.
6. Given points $A (-4, -5)$ and $B (6, -3)$. Find the coordinates of the midpoint of $AB$.
7. A point $P$ is at a distance of 3 from the center of a circle whose radius is 5. Find the product of the segments of any chord drawn through $P$.
8. In triangle $ABC$, a line parallel to base $AC$ cuts $AB$ at $D$ and $BC$ at $E$. If $BD = 6$, $DA = 2$ and $BC = 12$, find $BE$.
9. Corresponding sides of two similar triangles are in the ratio 1:4. If the area of the smaller triangle is 9, find the area of the larger triangle.
10. Angle $ABC$ is inscribed in circle $O$ and radii $OA$ and $OC$ are drawn. If angle $ABC = 40^\circ$, find the number of degrees in angle $AOC$.
11. The bases of a trapezoid are 4 and 16 and the altitude is 4. Find the area of this trapezoid.
12. In parallelogram $ABCD$, angle $A$ is represented by $x^\circ$ and angle $B$ by $(2x + 60)^\circ$. Find the number of degrees in angle $A$.
13. A vertical pole 30 feet high casts a shadow 16 feet long on level ground. Find to the nearest degree the angle of elevation of the sun.
14. Given the points $A (-2, 1)$ and $B (6, 7)$. Find the length of line segment $AB$.
15. The angle of a sector of a circle is $40^\circ$ and the area of the sector is $4\pi$. Find the radius of the circle.
16. Write an equation of the locus of points whose ordinates exceed twice their abscissas by 3.
17. The sum of the interior angles of a polygon is $1,800^\circ$. Find the number of sides of the polygon.

Directions (18-22): Indicate the correct completion for each of the following by writing the letter $a$, $b$, $c$ or $d$ on the line at the right.

18. Every triangle is divided into two equal triangles by $(a)$ a median $(b)$ an altitude $(c)$ the bisector of one of its angles $(d)$ any line from a vertex drawn to the opposite side.
19. The number of points which are at a given distance from a given line and also equidistant from two given points on the line is $(a)$ one $(b)$ two $(c)$ three $(d)$ four.
20. If a point is equidistant from the sides of a triangle, it must be the intersection of the three $(a)$ altitudes $(b)$ perpendicular bisectors of the sides $(c)$ medians $(d)$ angle bisectors.
21. Below are given four terms arranged in different orders. Which arrangement represents the sequence in which the definitions of these terms should be given?
(a) polygon, quadrilateral, rectangle, parallelogram (b) quadrilateral, polygon, parallelogram, rectangle (c) polygon, quadrilateral, parallelogram, rectangle (d) quadrilateral, polygon, rectangle, parallelogram

22. All members of the Math Club must have completed tenth year mathematics. Which statement expresses a conclusion that logically follows from the given statement? (a) If John has completed tenth year mathematics, he is a member of the Math Club. (b) If Mary has not completed tenth year mathematics, she is not a member of the Math Club. (c) If Tom is not a member of the Math Club, he has not completed tenth year mathematics. (d) If Anne is a member of the Math Club, she is an honor student in mathematics.

Directions (23-24): If the blank space in each statement below is replaced by the word always, sometimes or never, the resulting statement will be true. Select the word that will correctly complete each statement and write this word on the line at the right.

23. Two triangles are . . . congruent if they have a side and any two angles of one equal to the corresponding parts of the other.

24. If one angle of a scalene triangle is 60 degrees, the side opposite this angle is . . . the longest side of the triangle.

Directions (25): Leave all construction lines on the paper.

25. Divide line segment AB into three equal parts.

\[ \overline{A} \overline{E} \overline{B} \]

Part II

Answer three questions from this part.

26. Prove: A diameter perpendicular to a chord of a circle bisects the chord and its minor arc. [10]

27. Prove that if one leg of an isosceles triangle is the diameter of a circle, the circumference bisects the base. [10]

28. In the figure at the right, triangle MAB is inscribed in a circle. XY is tangent to the circle at point M. Chord CD is parallel to XY and intersects MA and MB at points E and K, respectively.

Prove: \[ \frac{ME}{MB} = \frac{MK}{MA} \] [10]

29. Prove: The area of a trapezoid is equal to one-half the product of the altitude and the sum of the bases. [10]

30. The coordinates of the vertices of triangle RST are R (0, 0), S (2a, 0) and T (a, b).
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a. Express in terms of \(a\) and \(b\) the length of \(RT\) and of \(ST\). \([6]\)

b. If the number of degrees in angle \(TRS\) is represented by \(n\), express in terms of \(n\) the number of degrees in angle \(RTS\). \([4]\)

*31. a. Using graph paper, draw the triangle whose vertices are \(A\) \((2, 3)\), \(B\) \((8, 3)\) and \(C\) \((6, 7)\). \([1]\)

b. Draw the locus of points equidistant from \(A\) and \(B\). \([1]\)

c. Write an equation of the line drawn in answer to part \(b\). \([2]\)

d. Write an equation of the line through \(A\) and \(C\). \([4]\)

e. Point \(P\) in line \(AC\) is equidistant from points \(A\) and \(B\). Find the coordinates of point \(P\). \([2]\)

*This question is based on one of the optional topics in the syllabus and may be used in place of any question in either part II or part III.

Part III

Answer two questions from this part. Show all work.

32. In the figure at the right, \(PA\) is a tangent and \(PAB\) is a secant to the circle. Angle \(P\) is represented by \(x^\circ\) and arcs \(AB\), \(BC\) and \(CA\) are represented by \((4x - 20)^\circ\), \((3x + 40)^\circ\) and \(y^\circ\), respectively.

a. In terms of \(x\) and \(y\), write a set of equations that can be used to solve for \(x\) and \(y\). \([4]\)

b. Solve the set of equations written in answer to part \(a\). \([5]\)

c. Find the number of degrees in arc \(BAC\). \([1]\)

33. Rectangle \(ABCD\) has the longer side \(AD\) as base and diagonal \(AC\). The perpendicular line from \(B\) to \(AC\) meets \(AC\) at \(E\). If \(BE = 12\) and \(EC\) exceeds \(AE\) by 7, find

a. \(AE\) and \(EC\) \([6]\)

b. the area of rectangle \(ABCD\) \([4]\)

34. The perimeter of a regular pentagon is 40.

a. Find to the nearest tenth the apothem of the polygon. \([7]\)

b. Using the result obtained in answer to part \(a\), find the area of the polygon. \([3]\)

35. Point \(A\) \((-5, 1)\) is the center of a circle, and point \(C\) \((-2, 7)\) lies on the circle. A straight line joins point \(B\) \((10, 1)\) to point \(C\). Using coordinate geometry, show that \(BC\) is tangent to the circle at point \(C\). \([10]\)