TENTH YEAR MATHEMATICS—JUNE 1959 (1)

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. Find a diagonal of a square whose side is 6.

2. A diagonal of a rectangle is 17 and the length is 15. Find the width of the rectangle.

3. In parallelogram $ABCD$, $AB = 10$, $AD = 6$ and angle $A = 30°$. Find the area of the parallelogram.

4. The area of a rhombus is 60 and one of its diagonals is 12. Find the other diagonal.

5. A secant and a tangent are drawn to a circle from an external point. The secant is 12 and its external segment is 3. Find the length of the tangent.

6. Given points A $(-4, -5)$ and B $(6, -3)$. Find the coordinates of the midpoint of AB.

7. A point P is at a distance of 3 from the center of a circle whose radius is 5. Find the product of the segments of any chord drawn through P.

8. In triangle ABC, a line parallel to base AC cuts AB at D and BC at E. If $BD = 6$, $DA = 2$ and $BC = 12$, find BE.

9. Corresponding sides of two similar triangles are in the ratio 1:4. If the area of the smaller triangle is 9, find the area of the larger triangle.

10. Angle ABC is inscribed in circle O and radii OA and OC are drawn. If angle $ABC = 40°$, find the number of degrees in angle AOC.

11. The bases of a trapezoid are 4 and 16 and the altitude is 4. Find the area of this trapezoid.

12. In parallelogram $ABCD$, angle A is represented by $x°$ and angle B by $(2x + 60)°$. Find the number of degrees in angle A.

13. A vertical pole 30 feet high casts a shadow 16 feet long on level ground. Find to the *nearest degree* the angle of elevation of the sun.

14. Given the points A $(-2, 1)$ and B $(6, 7)$. Find the length of line segment AB.

15. The angle of a sector of a circle is 40° and the area of the sector is 4π. Find the radius of the circle.

16. Write an equation of the locus of points whose ordinates exceed twice their abscissas by 3.

17. The sum of the interior angles of a polygon is 1,800°. Find the number of sides of the polygon.

Directions (18-22): Indicate the correct completion for *each* of the following by writing the letter *a, b, c* or *d* on the line at the right.

18. Every triangle is divided into two equal triangles by (*a*) a median (*b*) an altitude (*c*) the bisector of one of its angles (*d*) any line from a vertex drawn to the opposite side

19. The number of points which are at a given distance from a given line and also equidistant from two given points on the line is (*a*) one (*b*) two (*c*) three (*d*) four

20. If a point is equidistant from the sides of a triangle, it must be the intersection of the three (*a*) altitudes (*b*) perpendicular bisectors of the sides (*c*) medians (*d*) angle bisectors

21. Below are given four terms arranged in different orders. Which arrangement represents the sequence in which the definitions of these terms should be given?

(*a*) polygon, quadrilateral, rectangle, parallelogram (*b*) quadrilateral, polygon, parallelogram, rectangle (*c*) polygon, quadrilateral, parallelogram, rectangle (*d*) quadrilateral, polygon, rectangle, parallelogram

22. All members of the Math Club must have completed tenth year mathematics. Which statement expresses a conclusion that logically follows from the given statement? (*a*) If John has completed tenth year mathematics, he is a member of the Math Club. (*b*) If Mary has not completed tenth year mathematics, she is not a member of the Math Club. (*c*) If Tom is not a member of the Math Club, he has not completed tenth year mathematics. (*d*) If Anne is a member of the Math Club, she is an honor student in mathematics.

Directions (23-24): If the blank space in each statement below is replaced by the word *always, sometimes* or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

23. Two triangles are . . . congruent if they have a side and any two angles of one equal to the corresponding parts of the other.

24. If one angle of a scalene triangle is 60 degrees, the side opposite this angle is . . . the longest side of the triangle.

Directions (25): Leave all construction lines on the paper.

25. Divide line segment *AB* into *three* equal parts.

Part II

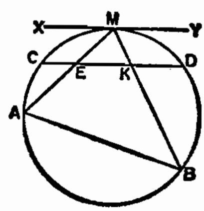
Answer three questions from this part.

26. Prove: A diameter perpendicular to a chord of a circle bisects the chord and its minor arc. [10]

27. Prove that if one leg of an isosceles triangle is the diameter of a circle, the circumference bisects the base. [10]

28. In the figure at the right, triangle *MAB* is inscribed in a circle. *XY* is tangent to the circle at point *M*. Chord *CD* is parallel to *XY* and intersects *MA* and *MB* at points *E* and *K*, respectively.

Prove: $\frac{ME}{MB} = \frac{MK}{MA}$ [10]



29. Prove: The area of a trapezoid is equal to one-half the product of the altitude and the sum of the bases. [10]

30. The coordinates of the vertices of triangle *RST* are *R* (0, 0), *S* (2*a*, 0) and *T* (*a*, *b*).

a. Express in terms of a and b the length of RT and of ST. [6]
b. If the number of degrees in angle TRS is represented by n, express in terms of n the number of degrees in angle RTS. [4]

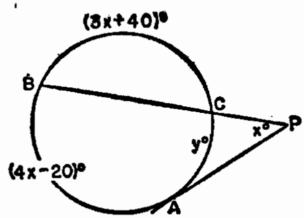
*31. *a.* Using graph paper, draw the triangle whose vertices are A (2, 3), B (8, 3) and C (6, 7). [1]
b. Draw the locus of points equidistant from A and B. [1]
c. Write an equation of the line drawn in answer to part *b.* [2]
d. Write an equation of the line through A and C. [4]
e. Point P in line AC is equidistant from points A and B. Find the coordinates of point P. [2]

* This question is based on one of the optional topics in the syllabus and may be used in place of any question in *either* part II or part III.

Part III

Answer two questions from this part. Show all work.

32. In the figure at the right, PA is a tangent and PCB is a secant to the circle. Angle P is represented by $x°$ and arcs AB, BC and CA are represented by $(4x - 20)°$, $(3x + 40)°$ and $y°$, respectively.



a. In terms of x and y, write a set of equations that can be used to solve for x and y. [4]
b. Solve the set of equations written in answer to part *a.* [5]
c. Find the number of degrees in arc BAC. [1]

33. Rectangle $ABCD$ has the longer side AD as base and diagonal AC. The perpendicular line from B to AC meets AC at E. If $BE = 12$ and EC exceeds AE by 7, find
a. AE and EC [6]
b. the area of rectangle $ABCD$ [4]

34. The perimeter of a regular pentagon is 40.
a. Find to the *nearest tenth* the apothem of the polygon. [7]
b. Using the result obtained in answer to part *a*, find the area of the polygon. [3]

35. Point A $(-5, 1)$ is the center of a circle, and point C $(-2, 7)$ lies on the circle. A straight line joins point B (10, 1) to point C. Using coordinate geometry, show that BC is tangent to the circle at point C. [10]