The University of the State of New York
318TH HIGH SCHOOL EXAMINATION
TENTH YEAR MATHEMATICS
Monday, June 15, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in tenth year mathematics, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II
Answer three questions from part II.

26 Prove: In a right triangle, the altitude is drawn upon the hypotenuse, (a) the two triangles thus formed are similar to the given triangle and (b) each leg of the given triangle is the mean proportional between the hypotenuse and the segment of the hypotenuse adjacent to that leg. \([3, 5]\)

27 In parallelogram \(ABCD\), side \(AB\) is shorter than side \(BC\). \(K\) is a point on \(BC\) such that \(DK\) bisects angle \(ADC\) and \(BK\) equals \(AB\).
   \(a\) Prove: \(CK = CD\) \([6]\)
   \(b\) Prove that \(K\) is the mid-point of \(BC\). \([4]\)

28 Prove: The area of a triangle is equal to one half the product of a side and the altitude drawn to that side. \([10]\)

29 In the figure at the right, \(HCB\) and \(HDA\) are secants to circle \(O\). Chords \(EB\) and \(EC\) intersect \(AH\) in \(F\) and \(G\) respectively.
   \(AB: BC: \text{Arc } CD = 3:2:1\).
   \(a\) Letting \(n\) equal the number of degrees in \(\text{arc } CD\), find in terms of \(n\) the number of degrees in arcs \(BC\) and \(AB\). \([1]\)
   \(b\) Prove that angle \(E\) equals angle \(H\). \([4]\)
   \(c\) Prove that triangle \(GEF\) is similar to triangle \(CGH\). \([3]\)
   \(d\) If \(CH\) is 12, \(EG\) is 10 and \(EF\) is 8, find \(GH\). \([2]\)

30 Prove: Two acute triangles are congruent if two sides and the altitude to one of these sides in one triangle are equal to the corresponding parts of the other triangle. \([10]\)
Part III

Answer two questions from part III.

31 Acute triangle $ABC$ is inscribed in circle $O$. The radius of the circle is 10 inches, and sides $AB$ and $BC$ are each 16 inches. $OE$ is drawn perpendicular to side $AB$, meeting $AB$ at $E$. $OA$, $OB$ and $OC$ are drawn.

a Find to the nearest degree angle $EOB$. [3]

b Using the result found in answer to a, find angle $AOB$ and obtuse angle $AOC$. [1, 3]

c Using a result found in answer to b, find to the nearest inch the distance from $O$ to chord $AC$. [3]

32 In trapezoid $ABCD$, base $AB$ is 8 inches and base $DC$ is 20 inches. Sides $DA$ and $CB$ are extended to meet at point $G$. The altitude of the trapezoid is 3 inches longer than the altitude to side $AB$ of triangle $GAB$.

a Using $x$ to represent the altitude from $G$ in triangle $GAB$, represent the corresponding altitude in triangle $GDC$. [2]

b Find the length of the altitude of the trapezoid. [6]

c Find the area of the trapezoid. [2]

33 a $A(2, 4)$, $B(10, -2)$ and $C(12, 9)$ are the vertices of triangle $ABC$. Show that triangle $ABC$ is isosceles. [4]

b In parallelogram $ABCD$, diagonals $AC$ and $BD$ meet in $E$. Side $AD = 25$, $AE = x + 3$, $CE = 3x - 5$, $BE = 3y - 6$ and $DE = 2y + 4$.

(1) Find $x$ and $y$. [2]

(2) Show that parallelogram $ABCD$ is a rhombus. [4]

*34 The vertices of triangle $ABC$ are $A(-4, -3)$, $B(4, 13)$ and $C(6, 7)$.

a Find the coordinates of $M$ and $N$, the mid-points of $AB$ and $BC$ respectively. [2]

b Show by using slopes that $MN$ is parallel to $AC$. [5]

c Write the equation of the line passing through $M$ and $N$. [3]

* This question is based upon one of the optional topics in the syllabus.
TENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil........................................ Name of school........................................

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. The vertex angle of an isosceles triangle contains 70°. Find the number of degrees in a base angle.

2. Two angles are supplementary, and one is 32° larger than the other. Find the number of degrees in the smaller angle.

3. Find the number of degrees in an angle of a regular pentagon.

4. Find the radius of a circle whose area is 36π.

5. A central angle of a circle whose radius is 18 contains 50°. Find the length of its arc. [Answer may be left in terms of π.]

6. Find the area of an equilateral triangle whose side is 8. [Answer may be left in radical form.]

7. The area of a rhombus is 72. If one diagonal is 16, find the other diagonal.

8. The altitude to the hypotenuse of a right triangle is 12. If one segment of the hypotenuse is 18, find the other segment.

9. A secant and a tangent are drawn to a circle from an external point. If the secant is 16 and its external segment is 4, find the tangent.

10. In triangle ABC, angle C equals 90°, angle A equals 71° and AC equals 20 inches. Find to the nearest inch the length of BC.

11. One side of a polygon is 2, and the corresponding side of a similar polygon is 3. Find the ratio of the areas of the two polygons.

12. Find the length of the line segment which joins the mid-points of two sides of a triangle whose third side is 10 inches.

13. A circle whose center is (7, 2) passes through the point (2, 14). Find the radius of the circle.

14. If the coordinates of A and B are (−5, 6) and (9, 2), find the coordinates of the mid-point of line segment AB.

[OVER]
15 All triangles whose vertices are (2, 2), (6, 2) and (x, 10), where x may have any value, are equal in area. Is this statement true or is it false? [Answer true or false.]

16 Write the equation of the locus of the centers of circles which are tangent to the line y = 8 and also tangent to the line y = 2.

Directions (17-22): Indicate the correct completion for each of the following by writing on the line at the right the letter a, b or c.

17 As the number of sides of a polygon increases, the number of degrees in the sum of the exterior angles (a) increases (b) decreases (c) remains the same 17.

18 A circle can always be circumscribed about any (a) parallelogram (b) rhombus (c) rectangle 18.

19 Chords $AB$ and $CD$ of a circle intersect in $E$. If arc $AD$ is $4m^\circ$ and arc $BC$ is $2m^\circ$, then angle $AED$ contains (a) $m^\circ$ (b) $2m^\circ$ (c) $3m^\circ$ 19.

20 Point C is 3 inches from given line $AB$. The number of points in $AB$ 5 inches from C is (a) 0 (b) 1 (c) 2 20.

21 If $A$, $B$, $C$ and $D$ are four consecutive points on a circle such that arc $AB$ equals arc $CD$, then chords $BC$ and $AD$ always (a) are equal (b) intersect (c) are parallel 21.

22 In triangle $ABC$, angle $C$ equals $60^\circ$ and $AB$ is greater than $AC$. Angle $B$ is (a) equal to $60^\circ$ (b) less than $60^\circ$ (c) greater than $60^\circ$ 22.

23 (1) An exterior angle of a triangle is equal to the sum of the nonadjacent interior angles.
   (2) An angle formed by two secants is measured by one half the difference of the intercepted arcs.
   (3) An inscribed angle is measured by one half the intercepted arc.
   (4) A central angle is measured by its intercepted arc.
Which one of the following represents the order in which the above statements should be arranged so that they form a logical sequence? (a) 4, 1, 2, 3 (b) 4, 1, 3, 2 (c) 4, 3, 1, 2 23.

24 The construction of tangents to circle $O$ from external point $B$ is shown at the right. Which one of the following statements is used to prove the construction? [Answer a or b.]
   a A tangent is perpendicular to a radius drawn to the point of contact.
   b A line perpendicular to a radius at its outer extremity is tangent to the circle. 24.

Directions (25): Leave all construction lines on your paper.

25 Divide line segment $AB$ into three equal parts. $A$–———————————–$B$