

The University of the State of New York

312TH HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, June 18, 1951 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in tenth year mathematics, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer two questions from part II.

26 Prove either *a* or *b*:

*a* An angle formed by two chords intersecting inside a circle is measured by one half the sum of the intercepted arcs. [10]

*b* An angle formed by two secants intersecting outside the circle is measured by one half the difference of the intercepted arcs. [10]

27 In triangles  $ABC$  and  $A'B'C'$ , side  $AB$  equals  $A'B'$ , side  $BC$  equals  $B'C'$  and median  $CM$  equals median  $C'M'$ .

Prove: *a* triangles  $MBC$  and  $M'B'C'$  are congruent [6]

*b* triangles  $ABC$  and  $A'B'C'$  are congruent [4]

28 The bisector of angle  $A$  of triangle  $ABC$  intersects  $BC$  at  $D$ .

Prove: *a* angle  $ADC$  is greater than angle  $DAC$  [7]

*b*  $AC$  is greater than  $DC$  [3]

Part III

Answer two questions from part III.

29 Answer both *a* and *b*:

*a* A line parallel to side  $AC$  of triangle  $ABC$  intersects  $AB$  at  $D$  and  $CB$  at  $E$ .  $BE = 20$  in.,  $EC = 15$  in. and  $BD$  is 3 in. longer than  $AD$ . Write an equation that can be used to find  $AD$ . Find  $AD$ . [2, 1]

*b* The hypotenuse of a right triangle is 20 and the altitude on the hypotenuse is 6. If  $x$  represents the shorter segment of the hypotenuse made by the altitude, write an equation that can be used to find  $x$ . Find  $x$  and then find the shorter leg of the right triangle. [Answer may be left in radical form.] [2, 3, 2]

30 Answer both *a* and *b*:

*a* Find the radius of the circle which is tangent to the  $x$ -axis and whose center is the point  $(10, 5)$ . Without constructing the circle, show that it passes through the point  $(13, 9)$ . [1, 4]

*b* The vertices of a triangle are the points  $(2, 3)$ ,  $(5, 8)$  and  $(4, 11)$ . Find the area of the triangle. [5]

31 Chord  $AB$  of a circle is 7.66 inches from the center and arc  $AB$  is  $80^\circ$ .

Find: *a* the radius of the circle [5]

*b* the area of the circle [Use  $\pi = 3.14$ .] [2]

*c* the length of minor arc  $AB$  [Answer may be left in terms of  $\pi$ .] [3]

[1]

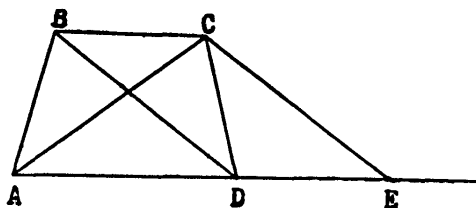
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Part IV

Answer one question from part IV.

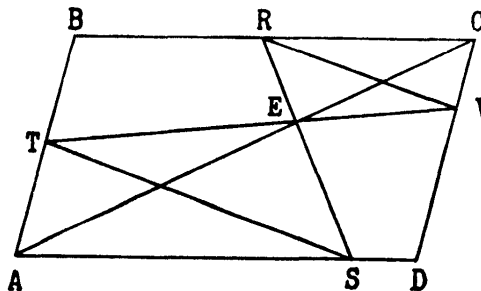
32 Given trapezoid  $ABCD$  with diagonal  $AC$  equal to diagonal  $BD$ . Through  $C$  a line is drawn parallel to  $BD$  intersecting base  $AD$  extended at  $E$ .

- Prove: a  $AC = CE$  [4]  
 b triangle  $ACD$  is congruent to triangle  $ABD$  [5]  
 c  $AB = DC$  [1]



33 Given parallelogram  $ABCD$  with  $E$  a point on diagonal  $AC$ . Lines through  $E$  intersect  $BC$  at  $R$ ,  $AD$  at  $S$ ,  $AB$  at  $T$  and  $CD$  at  $V$ .

- Prove: a  $AE:EC = TE:EV$  [3]  
 b  $AE:EC = SE:ER$  [2]  
 c triangle  $TES$  is similar to triangle  $REV$  [3]  
 d  $TS$  is parallel to  $RV$  [2]



\*34 Answer both  $a$  and  $b$ :

- a Prove that the slope  $m$  of the straight line through the points  $P_1(x_1, y_1)$  and  $P_2(x_2, y_2)$  is given by the formula  $m = \frac{y_1 - y_2}{x_1 - x_2}$  [7]

[Consider only the case in which  $m$  is positive and the points  $P_1$  and  $P_2$  are in the first quadrant.]

- b Find the equation of the straight line which passes through the origin and is parallel to the line through the points  $(2, 5)$  and  $(5, 6)$ . [3]

\* This question is based upon one of the optional topics in the syllabus.

TENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

- |   |         |
|---|---------|
| 1 Express the diagonal of a square in terms of its side $s$ .   | 1.....  |
| 2 Corresponding sides of two similar triangles are in the ratio 3:5. Find the ratio of their areas.   | 2.....  |
| 3 The diagonals of a rhombus are 6 and 12. Find its area.   | 3.....  |
| 4 In triangle $ABC$ side $AC$ is 8, side $AB$ is 4 and angle $A$ is $30^\circ$ . Find the area of the triangle.   | 4.....  |
| 5 Find the area of a trapezoid whose bases are 12 and 20 and whose altitude is 5.   | 5.....  |
| 6 The perimeter of a regular polygon is 40 and its apothem is 6. Find the area of the polygon.  | 6.....  |
| 7 The sides of a right triangle are 3, 4 and 5. Express as a common fraction the sine of the smallest angle.  | 7.....  |
| 8 A tangent and a secant are drawn to a circle from an external point. If the tangent is 12 and the external segment of the secant is 8, find the entire secant.  | 8.....  |
| 9 Point $P(x, 8)$ is 10 units from the origin. Find the positive value of $x$ .   | 9.....  |
| 10 The coordinates of the end points of line segment $AB$ are $A(3, 5)$ and $B(6, 9)$ . If $AB$ is extended its own length to point $C$ , find the coordinates of $C$ .   | 10..... |
| 11 Is the following statement true or is it false?<br>If the side of a square is the mean proportional between the base and the altitude of a parallelogram, the square and the parallelogram are equal.                                  | 11..... |
| 12 $AB$ is tangent to circle $O$ at $B$ and chord $BC$ is drawn forming an angle of $50^\circ$ . Find the number of degrees in arc $BC$ .   | 12..... |
| 13 Is statement $B$ the converse of statement $A$ ? [Answer <i>yes</i> or <i>no</i> .]<br>$A$ If two sides of a triangle are equal, the angles opposite these sides are equal.<br>$B$ The base angles of an isosceles triangle are equal. | 13..... |

Directions (questions 14–17) — If the blank space in each of the following statements is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

- |   |         |
|---|---------|
| 14 If two triangles have a side and any two angles of one equal to the corresponding parts of the other, the triangles are ... congruent. | 14..... |
| 15 An altitude of a triangle divides it into two triangles that are ... similar.  | 15..... |
| 16 The line joining the mid-points of the legs of a trapezoid divides it into two trapezoids which are ... equal.                         | 16..... |
| 17 In any circle if two chords are unequal, the longer chord is ... nearer the center of the circle.                                      | 17..... |

Directions (questions 18-24) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

18 The locus of points at a given distance from a given straight line is (a) a circle  
(b) one straight line (c) two parallel lines 18.....

19 If a median of a triangle is equal to one half of the side to which it is drawn, the triangle must be (a) isosceles (b) right (c) equiangular 19.....

20 If the point of intersection of the altitudes of a triangle is outside the triangle, the triangle is (a) acute (b) right (c) obtuse 20.....

21 If the diagonals of a parallelogram are equal, the parallelogram must be a (a) rectangle (b) square (c) rhombus 21.....

22 In *a*, *b* and *c* below, the same four terms are arranged in different orders. Which of these orders represents the sequence in which the *definitions* of these four terms should be given?

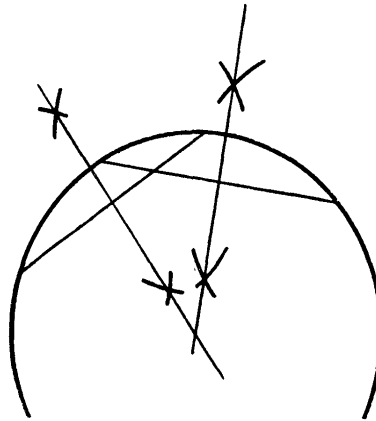
- a* polygon, triangle, right triangle, hypotenuse
- b* hypotenuse, polygon, right triangle, triangle
- c* triangle, polygon, hypotenuse, right triangle 22.....

23 In Eastview High School, only seniors can belong to the Alpha Society. Anne is not a senior. Therefore, she is not a member of the Alpha Society. This argument is an illustration of (a) indirect reasoning (b) reasoning from a converse (c) reasoning from an inverse (opposite) 23.....

24 To find the center of the circle of which the given arc is a part, the perpendicular bisectors of two chords of the arc are constructed as shown in the accompanying diagram.

Which of the following theorems is the one used to *prove* that the intersection of these bisectors is the required point?

- a* A diameter perpendicular to a chord bisects the chord.
- b* A diameter which bisects a chord, not a diameter, is perpendicular to the chord.
- c* The perpendicular bisector of a chord of a circle passes through the center of the circle.



25 Given circle *O* and point *P* on the circle. Construct a line tangent to the circle at *P*. 24.....

