The University of the State of New York

300TH HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Tuesday, June 17, 1947 - 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have

studied, (b) number of weeks and recitations a week in tenth year mathematics, (c) author of textbook

used.

The minimum time requirement is four or five recitations a week for a school year,

Part II

Answer two questions from part II.

26 Prove that the sum of the angles of a triangle is equal to a straight angle. [10]

27 Answer either a or b:

- a Prove that the area of a parallelogram is equal to the product of one side and the altitude drawn to that side. [10]
- b Prove that if in a right triangle the altitude is drawn upon the hypotenuse,

(1) The two triangles thus formed are similar to the given triangle

(2) Either leg of the given triangle is the mean proportional between the hypotenuse and the segment adjacent to that leg

28 The bisector of angle C of the inscribed triangle ABC intersects AB at F and the circle at K. Prove: AC : FC = KC : BC

29 The bisector of angle B of triangle ABC intersects AC in D. Prove that AB is greater than AD. [10]

Part III

Answer two questions from part III.

30 Answer both a and b:

a The angles of triangle ABC are represented by $4a - 8^{\circ}$, $5a - 10^{\circ}$, and $7a + 6^{\circ}$.

(1) Express the sum of the angles in terms of a. [1]

(2) Find the value of a. [2]

(3) What kind of triangle is ABC? [2]

b Diagonal AC of parallelogram ABCD intersects diagonal BD at O. AO is represented by 2x + 5 in. and CO by 4x - 13 in.

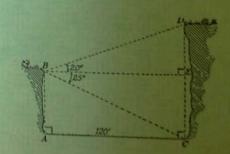
(1) Find the value of x and the length of AC.

(2) Diagonal DB is represented by 5x + 1 in. Find its length. [1] (3) What kind of parallelogram is ABCD? [1]

31 AB and CD are cliffs on opposite sides of a river 120 feet wide. From B the angle of elevation of D is 20° and the angle of depression of C is 25°. Find, correct to the nearest foot,

a The height of cliff AB

b The height of cliff CD





- Points of (2, 1), B(14, 1), C(14, 6) and D(2, 3) are the vertices of quadrilateral ABCD. Plot these points on graph paper and draw the quadrilateral. [4]
 - Find the lengths of AB, BC and diagonal AC. [3]
 - What kind of quadrilateral is ABCD1 [1]
 - d Fund the area of the quadrilateral. [2]

*33 Answer both a and b:

a Given the points A(2,3) and B(4,8)

(1) Find the slope of the straight line drawn through these points. [3]

(2) Find the distance from A to B. [Answer may be left in radical form.] [3]

& Tell whether each of the following statements is true or false: The straight line whose equation is y = x

(1) Passes through the origin

(2) Forms with the x axis an angle of 45°

(3) Is the locus of a point whose abscissa equals its ordinate [1]

(4) Is parallel to the line which passes through the points (0, 5) and (-1, 4)

* This question is based on one of the optional topics of the syllabus.

Part IV

Answer one question from part IV.

34 Read carefully statements a to d and in each case tell whether the reasoning is sound or unsound.

a Since it has been proved that a line which joins the midpoints of two sides of a triangle is parallel to the third side, it follows that a line which is parallel to one side of a triangle must pass through the midpoints of the other two sides. [1]

b If it is given that three parts of one triangle are equal to the corresponding three parts of another triangle, it follows that the triangles are congruent. [1]

c Since it has been proved that two acute angles whose sides are parallel are equal, it follows that two acute angles whose sides are not parallel are not equal. [1]

d Since a parallelogram is defined as a quadrilateral whose opposite sides are parallel, then if side AB of quadrilateral ABCD is given parallel to side CD and side BC is parallel to side AD, it follows that ABCD is a parallelogram. [1]

Each of the conclusions found in x, y, z below is reached by a type of reasoning similar to one of the types of reasoning used in a, b, c and d, above. Indicate in each case the statement, a, b, c or d, to which the reasoning corresponds most closely.

x All acids contain hydrogen. Since a given chemical compound is found to contain hydrogen,

it follows that this compound is an acid. [2]

y Young people who take an active part in outdoor sports usually are strong and healthy. It follows, therefore, that those who dislike activities of this kind and take no part in them are likely to be weak and sickly. [2]

z In Albany, New York, parking meters are not in operation on a business holiday. Therefore parking meters in that city are not in operation on January 1 of each year. [2]

35 CD is the altitude of triangle ABC and E is a point on CD such that $DE = \frac{1}{3}$ CD. A line through E parallel to AB intersects AC in K and BC in L. P, the midpoint of KL, is joined to Prove that: A, B

and C. Tiove Cor	[2]
$a \triangle CPK = \triangle CPL$	[4]
$b \triangle CAP = \triangle CBP$	10000
$ABP = \frac{1}{2} \triangle ABC$	[2]
$d \triangle PAC = \triangle PAB = \triangle PBC$	[2]
n Direc	[2]

Name of school.....

..Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 One angle of a right triangle is 30° and the leg opposite this angle is 4. Find the hypotenuse.

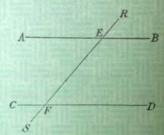
2 Find the altitude of the equilateral triangle whose side is 8. [Answer may be left in radical form.]

3 The side of a square is 8. Find the length of the diagonal. [Answer may be left in radical form.]

4 How many degrees are there in the sum of the exterior angles of any polygon, if one angle is taken at each vertex?

5 In triangle RST a line is drawn parallel to ST, intersecting RS in K and RT in L. If RK = 5, KS = 10 and RT = 27, find the length of RL.

6 Parallel lines AB and CD are cut by transversal RS. If $\angle REB = x^{\circ}$, and $\angle EFC = 2x^{\circ} + 30^{\circ}$, find the number of degrees in \(REB. \)



7 One of the base angles of an isosceles trapezoid is 40° and one of the equal sides is 5 feet. Find the altitude of the trapezoid correct to the nearest foot.

8 Two secants drawn from an external point to a circle intercept arcs of 70° and 120°. Find the number of degrees in the angle formed by the secants.

9 If the side of an equilateral triangle is 5, find the area of the triangle. [Answer may be left in radical form.]

10 The diagonals of a rhombus are 5 and 12. Find the area of the rhombus.

11 The areas of two similar triangles are in the ratio 1:4. Find the ratio of a pair of corresponding sides.

12 If the area of a circle is 36π , find its circumference. [Answer may be left in terms of π .

13 In circle O chord CD bisects chord AB at E. If CE = 3 and ED = 12, find the length of AE.

14 From a point A outside circle O, tangent AB and secant ACD are drawn. If AB is 6 and the external segment AC of the secant is 4, find the length of the entire secant.

15 What is the locus of points outside circle O and at a distance m from the circle?

16 Is statement b a converse of statement a? [Answer yes or no.] a Parallel chords intercept equal arcs on a circle.

b Two chords intercepting equal arcs on a circle are parallel.

TENTH YEAR MATHEMATICS	
17 Below are given four terms arranged in different orders. Which of these, a, b, c or d, represents the sequence in which the definitions of these four terms should be given?	
a Polygon, quadrilateral, rectangle, parallelogram	
o Polygon, rectangle, parallelogram, quadrilateral	
c Polygon, quadrilateral, parallelogram, rectangle	
d Polygon, parallelogram, rectangle, quadrilateral	17
Directions (questions 18-20) — If the blank in each statement is replace correctly complete each statement and write this word on the line at the right	ed by one of the words t the word that will
18 A median divides a triangle into two congruent triangles.	18
19 A trapezoid inscribed in a circle is isosceles.	19
20 If all the sides of a triangle are unequal, an angle bisector terminating	
in the opposite side is greater than the altitude drawn on that side.	20
21 The ordinate of a point on the line $2x + y = 7$ is 3. Find its abscissa.	21
22 The coordinates of point A are $(2, 3)$ and the coordinates of point B are $(6, 7)$. Find the coordinates of the midpoint of the line segment AB .	22
23 Write the equation of the line which is the locus of all points	22
equidistant from the points (2, 0) and (8, 0).	23
Directions (questions 24–25) — Leave all construction lines on your paper. 24 On line m find the point that is equidistant from points A and B .	
	·B
m m	
A	
25 Given triangle ABC, construct on base DE, corresponding to AC, a	triangle similar to
triangle ABC.	
Hiangie 112 9	
B	

C [4]

 \overline{D}

E