The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Wednesday, January 25, 1978 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

On page 9 you will find the "Tables of Natural Trigonometric Functions" which you may need to answer some questions in this examination. Fold this page along the perforations, and tear it off also slowly and carefully.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of \( \pi \) or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 In the accompanying diagram, circle \( O \) has radii \( OQ \) and \( OR \), chords \( PQ \) and \( PR \), and \( \angle QOR = 140^\circ \). Find \( \angle P \).

2 If the lengths of the legs of an isosceles triangle are represented by \( 3a + 2 \) and \( 5a - 14 \), find \( a \).

3 In the accompanying diagram, \( AB, BCD, \) and \( DE \) are each tangent to circle \( O \) at \( A, C, \) and \( E \), respectively. If \( AB = x \) and \( ED = y \), find the length of \( BD \) in terms of \( x \) and \( y \).

4 The measures of the angles of a triangle are in the ratio of 3:5:7. Find the number of degrees in the measure of the smallest angle of the triangle.

5 In parallelogram \( ABCD \) shown below, \( AD = 10 \) and \( \angle A = 30^\circ \). Find the length of the altitude drawn from \( D \) to \( AB \).

6 What is the area of a rhombus whose diagonals have lengths of 6 and 15?

7 Angle \( A \) and angle \( B \) are complementary. If the measure of angle \( A \) is 20 degrees more than the measure of angle \( B \), find \( \angle B \).

8 In the accompanying diagram, \( D \) is the midpoint of \( AB \) and \( E \) is the midpoint of \( AC \). If the perimeter of triangle \( ABC \) is 20, find the perimeter of triangle \( ADE \).

9 A circle is circumscribed about a right triangle. If the lengths of the legs of the triangle measure 12 and 16, find the length of the diameter of the circumscribed circle.

10 In a circle, an arc which measures \( 72^\circ \) has a length of 8 inches. Find, in inches, the circumference of the circle.

11 In \( \triangle ABC \), \( \angle C = 90^\circ, BC = 6, \) and \( AB = 9 \). Find, to the nearest degree, the measure of \( \angle B \).

12 In the accompanying diagram, lines \( \ell_1 \) and \( \ell_2 \) are parallel and \( \angle 1 = 70^\circ \). What must \( \angle 2 \) be so that lines \( \ell_3 \) and \( \ell_4 \) will be parallel?

13 Find the area of a regular polygon whose apothem is 6 and whose perimeter is 48.

Directions (14–29): Write in the space provided on the separate answer sheet the numeral preceding the expression that best completes each statement or answers each question.

14 The measure, in degrees, of each exterior angle of a regular 10-sided polygon is
   (1) 18
   (2) 36
   (3) 180
   (4) 360
15 In triangle $ABC$, a line parallel to $AB$ intersects $AC$ at $D$ and $BC$ at $E$. If the ratio of $CD$ to $CA$ is 2:3, then the ratio of $DE$ to $AB$ would be

- (1) $1:2$
- (2) $2:3$
- (3) $2:5$
- (4) $3:5$

16 Which set of integers can not be the lengths of the sides of a triangle?

- (1) $\{3,4,5\}$
- (2) $\{3,4,4\}$
- (3) $\{2,3,4\}$
- (4) $\{3,1,1\}$

17 A quadrilateral must be a parallelogram if two of its opposite sides are

- (1) congruent, only
- (2) parallel, only
- (3) congruent and parallel
- (4) parallel and the other two sides are congruent

18 A circle can not always be inscribed in which figure?

- (1) a right triangle
- (2) a trapezoid
- (3) a rhombus
- (4) a square

19 A line segment joining a vertex of a triangle to the midpoint of the side opposite is known as

- (1) an altitude
- (2) an angle bisector
- (3) a median
- (4) a perpendicular bisector

20 In the accompanying diagram, chords $AB$ and $CD$ of a circle intersect at point $E$. Chords $DB$ and $AC$ are drawn. $\triangle ACE$ and $\triangle DBE$ are always

- (1) right triangles
- (2) congruent triangles
- (3) equal in area
- (4) similar triangles

![Diagram](image)

21 In the accompanying diagram, triangle $ABC$ is a right triangle with right angle $C$. Altitude $\overline{CD}$ is drawn. If $AC = 10$ and $AB = 25$, then what is $AD$?

- (1) $\sqrt{250}$
- (2) $2.5$
- (3) $15$
- (4) $4$

22 The distance between the points $(2,3)$ and $(4,-2)$ is

- (1) $\sqrt{29}$
- (2) $\sqrt{7}$
- (3) $3$
- (4) $7$

23 If $JK > LM$ and $PQ > RS$, then which statement is not always true?

- (1) $JK - PQ > LM - RS$
- (2) $2PQ > 2RS$
- (3) $\frac{1}{2}JK > \frac{1}{2}LM$
- (4) $JK + PQ > LM + RS$

24 Point $A$ is 9 inches from a given straight line. How many points are both 4 inches from this line and 6 inches from point $A$?

- (1) 0
- (2) 2
- (3) 3
- (4) 4

25 If the points $A(2,k)$ and $B(6,10)$ determine a line whose slope is $\frac{1}{2}$, then $k$ is

- (1) $8$
- (2) $2$
- (3) $12$
- (4) $18$

26 Given: $\angle 1 \equiv \angle 2$. In order to prove by the indirect method that $\overline{AB} \parallel \overline{CD}$, what must be assumed?

![Diagram](image)

- (1) $\angle 1 \equiv \angle 2$
- (2) $\overline{AB} \parallel \overline{CD}$
- (3) $\angle 1$ not congruent to $\angle 2$
- (4) $\overline{AB}$ not parallel to $\overline{CD}$

27 The endpoints of a diameter of a circle are $(-3,8)$ and $(8,3)$. The coordinates of the center of the circle are

- (1) $(5,11)$
- (2) $(\frac{11}{2}, \frac{5}{2})$
- (3) $(\frac{5}{2}, \frac{11}{2})$
- (4) $(\frac{11}{2}, -\frac{5}{2})$

28 What is the area of a square whose diagonal has a length of 8?

- (1) $64\sqrt{2}$
- (2) $64$
- (3) $32\sqrt{2}$
- (4) $32$
29 The accompanying diagram shows the construction used for dividing $AB$ into three congruent parts. On which theorem is this based?

(1) The corresponding sides of congruent triangles are congruent.
(2) If a line segment joins the midpoints of two sides of a triangle, it is parallel to the third side.
(3) If three or more parallel lines cut off congruent segments on one transversal, they cut off congruent segments on any other transversal.
(4) If two lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the lines are parallel.

Directions (30): Leave all construction lines on the answer sheet.

30 On the answer sheet, construct the altitude from $B$ to $AC$ of triangle $ABC$. 
Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove either a or b, but not both. \[10\]

a If two sides of a triangle are congruent, the angles opposite these sides are congruent.

OR

b The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs.

32 In the accompanying diagram, ABCD is a parallelogram, AMD, DNC, APB, and BQC. MN and PQ are drawn and \( \angle 1 \equiv \angle 2 \).

Prove: \( MN \times BQ = PQ \times MD \) \[10\]

33 Given: \( \triangle BCD \), \( \overline{BC} \equiv \overline{BD} \); points A and E are exterior to \( \triangle BCD \). \( \overline{AB} \) and \( \overline{BE} \) are drawn so that \( \overline{AB} \equiv \overline{BE} \); \( \overline{AC} \) and \( \overline{DE} \) are drawn; \( \angle ABD \equiv \angle EBC \).

\( a \) Prove: \( \triangle ABC \equiv \triangle EBD \) \[5\]

\( b \) Prove: area polygon \( ABDC = \) area polygon \( BEDC \) \[5\]

34 In the accompanying diagram, points A, B, C, and D are located consecutively on a circle so that \( m\overline{AB} > m\overline{AD} \) and \( \overline{DC} \parallel \overline{AB} \). Chords \( \overline{AD} \) and \( \overline{DB} \) are drawn.

Prove: \( m\angle ADB > m\angle BDC \) \[10\]

35 In the accompanying diagram, \( \overline{ABC} \) and \( \overline{ADE} \) are secants, \( \overline{DC} \) and \( \overline{BE} \) intersect at F, \( m\angle E = 160 \), \( m\overline{BD} = 80 \), and \( \overline{BC} \equiv \overline{ED} \).

\( a \) Find \( m\angle BEC \). \[2\]

\( b \) Find \( m\angle CFE \). \[2\]

\( c \) Find \( m\angle CAE \). \[2\]

\( d \) Find \( m\angle CBE \). \[2\]

\( e \) Find \( m\angle EBA \). \[2\]

36 In rhombus ABCD, diagonals \( \overline{AC} \) and \( \overline{BD} \) are drawn, each side measures 20, and the measure of angle DAB is 52°.

\( a \) Find, to the nearest integer, the length of diagonal \( \overline{AC} \). \[4\]

\( b \) Find, to the nearest integer, the length of diagonal \( \overline{BD} \). \[4\]

\( c \) Using the results from parts \( a \) and \( b \), find the area of the rhombus. \[3\]

37 The vertices of \( \triangle ABC \) are \( A(-4,3) \), \( B(1,8) \), and \( C(6,3) \).

\( a \) Show that \( \triangle ABC \) is isosceles and state a reason for your conclusion. \[4\]

\( b \) Show that \( \triangle ABC \) is also a right triangle and state a reason for your conclusion. \[4\]

\( c \) Find the area of \( \triangle ABC \). \[3\]
# Tables of Natural Trigonometric Functions

(For use with 9th and 10th Year Mathematics Regents Examinations)

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[9]

[OVER]

Math. 10–Jan. '78
Your answers to Part I should be recorded on this answer sheet.

Part I
Answer all questions in this part.

1. .................................................. 21. ..................................................
2. .................................................. 22. ..................................................
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8. .................................................. 28. ..................................................
9. .................................................. 29. ..................................................
10. .................................................. 30. Answer question 30 on the other side of this sheet.
Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

__________________________
Signature

Math. 10-Jan. '78

[12]

164
FOR TEACHERS ONLY

SCORING KEY

TENTH YEAR MATHEMATICS

Wednesday, January 25, 1978 — 1:15 to 4:15 p.m., only

Use only red ink or red pencil in rating Regents papers. Do not attempt to correct the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 14–29, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

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Part II

Please refer to the Department’s pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(35) a 60 \[2\]

\[\begin{array}{ll}
 b & 120 \ [2] \\
 c & 40 \ [2] \\
 d & 80 \ [2] \\
 e & 100 \ [2]
\end{array}\]

(36) a 36 \[4\]

\[\begin{array}{ll}
 b & 18 \ [4] \\
 c & 324 \ [2]
\end{array}\]

(37) c 25 \[2\]