SPHERIC TRIGONOMETRY

*Group I*

Answer three questions from this group.

1. In any right spheric triangle, in which all parts except the right angle \( C \) are less than 90°, prove:
   
   \[
   \sin a = \sin c \sin A \quad \text{[Give geometric proof.]} \quad [8]
   \]
   \[
   \cos c = \cot A \cot B \quad [7]
   \]

2. Show that in a right spheric triangle each leg and the opposite angle are always of the same species, that is, are always in the same quadrant. \([15]\)

3. In any right spheric triangle prove the following identities:
   
   \[
   \sin b = \cos c \tan a \tan B \quad [8]
   \]
   \[
   \sin \pi A = \cos^2 B + \sin^2 a \sin^2 B \quad [7]
   \]

4. Starting with the formula
   
   \[
   \cos a = \cos b \cos c + \sin b \sin c \cos A,
   \]
   
   derive the formula
   
   \[
   \sin^2 \frac{A}{2} = \frac{\sin \pi (s - b) \sin (s - c)}{\sin b \sin c} \quad [15]
   \]

*Group II*

Answer one question from this group.

5. Solve and check the quadrant spheric triangle in which
   
   \[
   a = 122^\circ 40', \ C = 65^\circ 14', \ c = 98^\circ \quad [12, 3]
   \]

6. Solve and check the isosceles spheric triangle in which
   
   \[
   a = 55^\circ 24', \ b = 55^\circ 24', \ c = 70^\circ 52' \quad [11, 4]
   \]

*Group III—concluded*

Answer two questions from this group.

7. Solve the spheric triangle in which \( a = 130^\circ 14', \ b = 59^\circ 21', \ c = 98^\circ 16' \quad [29]\)

8. Find the shortest distance in nautical miles between Boston (42° 21' N., 71° 3' W.) and Paris (48° 36' N., 2° 31' E.). [A nautical mile is the length of 1' of the arc of a great circle on the earth's surface.] \([29]\)

9. Each lateral face of a regular square pyramid makes an angle of 81° with the base. What are the face angles at a corner of the base of the pyramid? \([29]\)