The University of the State of New York

310TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, August 24, 1950 — 12 m. to 3 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in solid geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1950 or number and length in minutes of lessons taken in the summer of 1950 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1950 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

21 Prove that if two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other. [10]

22 Two points A and B are on the same side of plane P. A line from A perpendicular to P intersects P at R. AR is extended its own length to A'. $A'B$ is drawn and intersects P at M.

a Prove that $AM + MB = A'B$ [4]

b Let M' be any point in P other than M.
Prove that $AM' + M'B > AM + MB$ [6]

23 Prove that the sum of the angles of a spherical triangle is greater than 180° and less than 540°. [10]

24 Given two skew lines r and s.

a Show how to pass a plane P through r and a plane Q through s so that P and Q will be parallel. [6]

b Prove that P is parallel to Q. [3]

c Can there be more than one pair of such planes? [1]

Part III

Answer three questions from part III.

25 A zone and a triangle are drawn on a sphere whose radius is 12″. The area of the triangle is equal to the area of the zone. The angles of the triangle are 96°, 84° and 80°. Find:

a the spherical excess of the triangle [2]

b the area of the triangle in square inches [Answer may be left in terms of π.] [4]

c the altitude of the zone to the *nearest tenth* of an inch. [4]

26 A storage bin has the form of a frustum of a quadrangular pyramid. The lower base is a rectangle 2′ 3″ by 3′, the upper base is a rectangle 3′ 9″ by 5′ and the depth is 4′. Find to the *nearest bushel* the capacity of the bin. [$V = \frac{h}{3}(B + B' + \sqrt{BB'}$ and 1 bu. = approximately $1\frac{1}{4}$ cu. ft.] [10]

27 The altitude to the base of an isosceles triangle is h, one of its base angles is θ and the volume of the solid formed by revolving the triangle through 180° about its altitude as an axis is V.

a Show that $h = \sqrt[3]{\dfrac{3V \tan^2 \theta}{\pi}}$ [5]

b Using logarithms, find h to the *nearest tenth* if $V = 50$ and $\theta = 75°$. [Use $\pi = 3.14$.] [5]

28 A cube is inscribed in a sphere whose diameter is d.

a Express the volume of the cube in terms of d. [5]

b Show that the volume of the sphere is approximately 2.7 times the volume of the cube. [5]

Fill in the following lines:

Name of pupil ....Name of school ....

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed.

Directions (questions 1–12) — Write the answer to *each* question on the line at the right.

1 Find the lateral area of a regular triangular pyramid whose base edge is 10 and whose slant height is 12. 1...............

2 The lateral area of a frustum of a cone of revolution is 42π square inches and the radii of its bases are 5 inches and 2 inches. Find its slant height. 2...............

3 The radius of the base of a right circular cylinder is 3 and its altitude is 7. Find its *total* area. [Answer may be left in terms of π.] 3...............

4 The radius of the base of a right circular cylinder is r, its altitude is $5r$ and its volume is 40π. Find r. 4...............

5 Find the number of degrees in the angle of a lune if its area is $\frac{1}{36}$ of the area of the sphere on which it is drawn. 5...............

6 Corresponding altitudes of two similar parallelepipeds are in the ratio $3:4$. Find the ratio of the volume of the smaller parallelepiped to the volume of the larger. 6...............

7 Express the *total* area of a regular octahedron in terms of its edge e. [Answer may be left in radical form.] 7...............

8 A plane parallel to the base of a pyramid forms a section whose area is 20 square inches. If the base of the pyramid is 180 square inches and its altitude is 12 inches, how far is the plane from the vertex of the pyramid? 8...............

9 A line segment is 12 inches in length. If its projection on a plane is 9.7 inches, find to the *nearest degree* its inclination to the plane. 9...............

10 The area of a small circle of a sphere is 9π square inches and the plane of the circle is 4 inches from the center of the sphere. Find the radius of the sphere. 10...............

11 In spherical triangle ABC, $\angle A = \angle B = 90°$ and side $AB = 100°$. How many degrees are there in $\angle C$? 11...............

12 How many points are there which are equidistant from all points on a given circle and also at a given distance, d, from the plane of the circle? 12...............

Directions (questions 13–16) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b*, or *c*.

13 The locus of points equidistant from the faces of a dihedral angle and also equidistant from two points on the edge of the angle is a (*a*) point (*b*) line (*c*) plane 13......

14 The volumes of two prisms which have equal bases are to each other as (*a*) the cubes of their altitudes (*b*) the squares of their altitudes (*c*) their altitudes 14......

15 Plane P intersects plane Q. If line r is perpendicular to P and line s is perpendicular to Q, then r and s (*a*) must intersect (*b*) may intersect (*c*) may be parallel 15......

16 Plane P intersects plane Q. If line r is parallel to P, then r (*a*) must intersect Q (*b*) must be parallel to Q (*c*) may be parallel to Q 16......

Directions (questions 17–20) — In *each* of the following, if the statement is *always* true, write *true* on the line at the right; if it is *not always* true, write *false.*

17 A right section of a circular cylinder is a circle. 17................

18 Through a given line only one plane can be passed perpendicular to a given plane. 18................

19 If two face angles of a trihedral angle are 140° and 100°, the third face angle must be greater than 40° and must be less than 120°. 19................

20 The polar triangle of an isosceles spherical triangle is isosceles. 20................