

The University of the State of New York

301ST HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Wednesday, August 20, 1947 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in solid geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1947 or number and length in minutes of lessons taken in the summer of 1947 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (d) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement, the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1947 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

21 Prove that if a point on a sphere is at a quadrant's distance from each of two other points on the sphere, not the extremities of a diameter, it is the pole of the great circle through these points. [10]

22 Show that any two of the following are true:

a A zone of altitude h is drawn on a sphere whose radius is r . A lune on the same sphere has the same area as the zone. If θ is the angle of the lune, show that

$$\theta = \frac{180^\circ \times h}{r} \quad [5]$$

b If the radius r of a sphere is increased by x , then the area of the sphere is increased by $4\pi x (2r + x)$ [5]

c If the sides of a spherical triangle are x° , y° and z° , then the spherical excess of its polar triangle is $360^\circ - (x^\circ + y^\circ + z^\circ)$ [5]

23 Prove that if two angles not in the same plane have their sides respectively parallel and extending in the same direction from their vertices, the angles are equal. [10]

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Part III

Answer three questions from part III.

24 A steel casting has the form of a regular square pyramid whose base edge is 6 inches and whose height is 9 inches. A hole, 1 inch in diameter, is bored into the casting at the center of its base to a depth of 6 inches. Allowing .28 pound per cubic inch, find the weight of the casting after the hole is bored. [Use $\pi = 3.14$ and give your answer correct to the *nearest tenth of a pound.*] [10]

25 a A metal cube with edge e is to be melted and cast into spherical shot of diameter d . Making no allowance for waste, show that the number N of such shot is given by the formula

$$N = \frac{6e^3}{\pi d^3} \quad [4]$$

b Using the formula given in part a, find N if $e = 2.5$ inches and $d = .25$ inch. Give your answer to the *nearest hundred shot.* [6]

26 The altitude on the base of an isosceles triangle is h and one of the base angles is θ .

a Show that the volume of the solid generated by revolving the triangle through 180° about

$$h \text{ as an axis is given by the formula } V = \frac{\pi h^3}{3 \tan^2 \theta} \quad [6]$$

b Find V , correct to the *nearest integer*, if $h = 6$ and $\theta = 22^\circ$ [4]

27 The angles of a spherical triangle are in the ratio 3 : 4 : 5. The area of the triangle is 77 and the radius of the sphere is 7. Find the angles of the triangle. [Use $\pi = \frac{22}{7}$] [10]

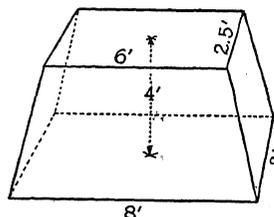
*28 a Tell whether *each* of the following statements is *true* or *false*:

- (1) The bases of a prismatoid are similar polygons. [1]
- (2) The bases of a prismatoid may be congruent. [1]
- (3) A frustum of a pyramid is a prismatoid. [1]
- (4) A lateral face of a prismatoid may be a triangle, a parallelogram or a trapezoid. [1]

b The base of a monument has the form of a prismatoid as shown in the figure. The lower base is a rectangle 8 feet by 3 feet, the upper base is a rectangle 6 feet by 2.5 feet and the height is 4 feet. Using the formula

$$V = \frac{h}{6} (B + B' + 4M), \text{ find,}$$

correct to the *nearest cubic foot*, the volume of the base of the monument. [6]



* This question is based on one of the optional topics in the syllabus.

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Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-4) — Write on the line at the right the word which, when inserted in the corresponding blank, will make the statement true.

- 1 Through a given point, one and only one ... can be drawn perpendicular to a given plane. 1.....
- 2 Through a given point outside a given plane, one and only one ... can be drawn parallel to the given plane. 2.....
- 3 If two of the face angles of a trihedral angle are 100° and 120° , the third face angle is greater than 20° and less than ... and may have any value between these limits. 3.....
- 4 The projection of parallelogram $ABCD$ on plane M is a parallelogram if the plane of $ABCD$ is *not* ... to M . 4.....

Directions (questions 5-8) — In *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

- 5 Through a given line which is oblique to a given plane, only one plane can be passed perpendicular to the given plane. 5.....
- 6 Through a given point, only one plane can be passed parallel to any two given lines. 6.....
- 7 The section formed by a plane cutting all the elements of a circular cone is a circle. 7.....
- 8 The locus of points equally distant from two parallel planes and also equally distant from two given points in one of these planes is a line. 8.....

Directions (questions 9-11) — In *each* of the following, *one* of the given choices will make the statement *false*. Indicate this choice by writing on the line at the right the letter a , b or c .

- 9 If a pyramid is cut by a plane parallel to its base, (a) the lateral edges of the pyramid are divided proportionally, (b) the section thus formed is similar to the base, (c) the area of the section is to the base as its distance from the vertex is to the altitude of the pyramid 9.....
- 10 A sphere can be circumscribed about (a) any cube, (b) any triangular pyramid (c) any parallelepiped 10.....
- 11 The locus of points at a given distance from plane M and also at a given distance from point P in the plane may be (a) one circle, (b) two circles, (c) two points 11.....
- 12 Find the lateral area of a right circular cone whose slant height is 10 and whose radius is 6. [Answer may be left in terms of π .] 12.....
- 13 Express the diagonal of a cube in terms of its edge e . 13.....
- 14 Find the total area of a right circular cylinder whose radius is 2 and whose altitude is 4. [Answer may be left in terms of π .] 14.....
- 15 The radii of the bases of a frustum of a right circular cone are 12 and 8 and the slant height of the frustum is 5. Find the lateral area. [Answer may be left in terms of π .] 15.....

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- 16 What fractional part of the surface of the earth is included between the meridians of 10° East longitude and 5° West longitude? 16.....
- 17 A lateral edge of an oblique prism is 8 and a right section is a triangle whose sides are 3, 4 and 5. Find the volume of the prism. 17.....
- 18 The volumes of two similar prisms are in the ratio 27:64 and the total area of the smaller prism is 45 sq. in. Find the total area of the larger prism. 18.....
- 19 A point is 8 inches from each face of a dihedral angle whose plane angle is 60° . How far is this point from the edge of the dihedral angle? 19.....
- 20 The inclination of a line segment to a plane is 38° and the projection of the segment on the plane is 39.4 inches. Find the length of the segment. 20.....