

The University of the State of New York

249TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Wednesday, August 20, 1930 — 12 m. to 3 p. m., only

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Instructions

*Do not open this sheet until the signal is given.*

*Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.*

*Part I is to be done first and the maximum time to be allowed for this part is one hour.*

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one hour period.

## Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions—Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 Two lines not in the same plane can not be perpendicular to the same .... *Ans.....*

2 The locus of points equidistant from the ceiling and floor of a rectangular room and  $x$  feet from the front wall is a .... *Ans.....*

3 If a line segment is not parallel to a plane, it is ... than its projection on the plane. *Ans.....*

4 The difference between two face angles of a trihedral angle is ... than the third face angle. *Ans.....*

5 If the diagonal of a cube is  $12\sqrt{3}$  inches, its volume is ... cubic inches. *Ans.....*

6 If a triangular pyramid always has the same height, no matter on what face it rests, its faces are ... triangles. *Ans.....*

7 Two right circular cones have equal volumes; if the ratio of their altitudes is 4:1, the ratio of their radii is .... *Ans.....*

8 The lateral area of a regular square pyramid is 48 square feet; if the area of the base is 36 square feet, the lateral edge is ... feet. *Ans.....*

9 An element of a right circular cone is 6 inches long and makes an angle of  $60^\circ$  with the base; its lateral area in terms of  $\pi$  is ... square inches. *Ans.....*

10 The formula for the volume of any pyramid is  $V = \dots$  *Ans.....*

11 If a plane parallel to the base of a pyramid forms a section whose area is  $\frac{1}{4}$  the area of the base, the plane will divide the corresponding altitude into two segments whose ratio is .... *Ans.....*

12 The weight of 1200 lead balls 1 inch in diameter is the same as the weight of ... lead balls 2 inches in diameter. *Ans.....*

13 Two sides of a given spheric triangle are  $60^\circ$  and  $50^\circ$ ; if the sum of the angles of the polar triangle is  $350^\circ$ , the third side of the given triangle contains ... degrees. *Ans.....*

14 If the ratio of the areas of two spheres is 4:9, the ratio of their volumes is .... *Ans.....*

15 If the lateral area of a right circular cylinder is equal to the sum of the areas of its bases, the altitude of the cylinder is equal to the ... of the base. *Ans.....*

16 A sphere whose radius is 13 inches is cut by a plane 12 inches from the center of the sphere; the radius of the small circle thus formed is ... inches.

Ans.....

17 If the area of a lune is  $\frac{1}{4}$  the surface of its sphere, the angle of the lune is ... degrees.

Ans.....

18-20 If an isosceles trapezoid with bases 4 inches and 8 inches and altitude 3 inches is rotated about the shorter base as an axis, the volume of the solid generated is equal to the volume of a (a) ... diminished by twice the volume of a (b) ... of which the altitude is (c) .... inches.

Ans. a.....

b.....

c.....

**SOLID GEOMETRY**  
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Write at top of first page of answer paper (a) names of schools where you have studied, (b) number of weeks and recitations a week in solid geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1930.

The minimum time requirement previous to entering summer high school is five recitations a week for half a school year, or the equivalent.

For those pupils who have met the time requirement previous to entering summer high school the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1930 is required.

Name the author of the textbook you have used in your study of solid geometry.

**Part II**

*Answer five questions from part II, including three questions from group I and two questions from group II.*

**Group I**

**Answer three questions from this group.**

21 Prove that if two angles not in the same plane have their sides respectively parallel and extending in the same direction from their vertices, they are equal and their planes are parallel. [12]

22 Prove that if a line in one face of a dihedral angle is parallel to a line in the other face, each is parallel to the edge of the angle. [12]

23 It is desired to find the locus of all points in space (1) equidistant from the vertices of a given triangle  $ABC$  and at the same time (2) at a given distance  $d$  from side  $AB$ .

a Describe the nature of each of the loci (1) and (2). [8]

b In general, what will be the required locus? [2]

c Under what condition would there be no locus? [2]

24 a Given a plane  $MN$  and a point  $A$  not in  $MN$ ; show how you would construct  $AB$ , a perpendicular to  $MN$ . [Proof not required] [6]

b Assuming  $AB$  drawn, explain how you would construct a plane through  $A$  making an angle of  $45^\circ$  with  $MN$ . [6]

**Group II**

**Answer two questions from this group.**

*Leave all work on the paper; merely writing the answers is not sufficient. Irrational results should be left in the form of  $\pi$  and radicals unless otherwise stated.*

25 A vessel in the form of a right circular cylinder 8 inches in diameter is partly filled with water. When 100 balls, equal in size, are dropped into the cylinder, the level of the water rises 8 inches. If all of the balls are completely immersed in the water, find the diameter of each ball. [12]

26 a Find the number of square feet in the area of a spheric triangle if its angles are  $120^\circ$ ,  $80^\circ$  and  $85^\circ$ , and the radius of the sphere is 20 feet. [6]

b How many cubic inches of mahogany will be required to veneer the top of a table in the shape of a regular hexagon, each side of which measures 2 feet, the veneer being  $\frac{1}{8}$  inch thick? [6]

27 A cone whose slant height is equal to the diameter of its base is inscribed in a given sphere and a similar cone is circumscribed about the same sphere. Find the ratio of the volumes of the two cones. [12]