The volume of a sphere is \(36\pi\) cubic inches. Find the number of inches in the radius of the sphere.

2. The lateral area of a certain cylinder of revolution is \(2\pi r^2\) where \(r\) is the radius of the base. What is the ratio of the height of the cylinder to the diameter of its base?

3. Two similar pyramids have heights of 2 inches and 6 inches, respectively. If the volume of the first is 10 cubic inches, find the number of cubic inches in the volume of the second pyramid.

4. Triangle \(RST\) is inscribed in circle \(O\) whose diameter \(RT\) is 24. Line segment \(OQ\) is perpendicular to the plane of the circle. If \(OQ\) is 5, find \(QS\).

5. Each lateral face of a regular square pyramid makes a 60° angle with the base. If the base edge is represented by \(m\), express the total area of the pyramid in terms of \(m\).

6. A rectangular solid has dimensions 3, 5 and 6. What is the length of a diagonal of the solid?

7. The lateral edge of a frustum of a regular triangular pyramid is 10 inches. If an upper base edge is 5 inches and a lower base edge is 17 inches, find the number of square inches in the lateral area of the frustum.

8. What is the locus of points which are equidistant from the faces of a trihedral angle?
9 The radius of a sphere is 12 inches and the radius of a small circle on this sphere is 8 inches. Find the distance in inches from the center of the sphere to the plane of the small circle.

10 Find the total area of a right circular cylinder circumscribed about a sphere whose radius is 5.

11 The angles of a spherical triangle are 50°, 125° and 104°. Find the number of degrees in the perimeter of its polar triangle.

12 The altitude of a zone is equal to \( \frac{1}{2} \) the radius, \( r \), of the sphere on which it is drawn. Express in terms of \( r \) the area of the zone.

13 Corresponding edges of two similar solids are in the ratio 3 : 5. If the lateral area of the smaller solid is 72, find the lateral area of the larger solid.

14 A pyramid and a prism have equal altitudes. The area of the base of the pyramid is 4 times the area of the base of the prism. What is the numerical value of the ratio of the volume of the pyramid to the volume of the prism?

15 Point \( P \) is equidistant from the faces of a 48° dihedral angle and 6 inches from the edge of the angle. Find to the nearest tenth the number of inches from \( P \) to either face.

16 Find in spherical degrees the area of an equilateral spherical triangle, each of whose angles is 70°.

17 The slant height \( l \) of a certain right circular cone is equal to the diameter of the base. A sphere is inscribed in the cone. Express in terms of \( l \) the radius of this sphere.

Directions (18–26): Write on the line at the right of each of the following the number preceding the expression that best completes the statement.

18 Two face angles of a trihedral angle are 125° and 65°. The third face angle may be
   (1) 170°
   (2) 60°
   (3) 190°
   (4) 90°

19 If two planes \( R \) and \( S \) make equal angles with a third plane, then \( R \) and \( S \)
   (1) are always perpendicular to the third plane
   (2) are always parallel to each other
   (3) may intersect each other
   (4) must intersect each other.

20 The total number of lines which can be drawn obliquely to a plane from an external point so as to make equal angles with the plane is
   (1) infinite
   (2) 2
   (3) 3
   (4) 4
21 Plane \( M \) and plane \( N \) are parallel to line \( AB \). If \( M \) intersects \( N \) in \( QR \), then
\[(1) \text{ are parallel} \quad (3) \text{ are skew} \]
\[(2) \text{ may intersect} \quad (4) \text{ may coincide} \]

22 It is not true that the diagonals of a cube
\[(1) \text{ are equal} \quad (3) \text{ bisect each other} \]
\[(2) \text{ are perpendicular} \quad (4) \text{ intersect in a point which is equidistant from the faces} \]

23 If a regular polyhedron has 12 edges and 6 vertices, the number of faces is
\[(1) \quad 12 \quad (3) \quad 6 \]
\[(2) \quad 8 \quad (4) \quad 4 \]

24 If a line intersects a plane and is not perpendicular to the plane, then this line is perpendicular to
\[(1) \text{ only one line in the given plane} \quad (3) \text{ every line in the given plane} \]
\[(2) \text{ two lines in the given plane} \quad (4) \text{ no line in the given plane} \]

25 The projection of a straight line segment upon a plane can not be a straight line segment when the given segment
\[(1) \text{ intersects the plane at a } 45^\circ \text{ angle} \quad (3) \text{ is perpendicular to the plane but does not intersect the plane} \]
\[(2) \text{ is oblique to the plane but does not intersect the plane} \quad (4) \text{ is parallel to the plane} \]

26 The total number of lines equidistant from two intersecting planes and also at
\[(1) \quad 1 \quad (3) \text{ infinite} \]
\[(2) \quad 2 \quad (4) \quad 4 \]

Directions (27–30): If the blank space in each statement below is replaced by the word always, sometimes (but not always) or never, the resulting statement will be true. Select the word that will correctly complete each statement and write this word on the line at the right.

27 Two spherical triangles which are symmetric to a third triangle are ... congruent to each other.

28 If two great circles are drawn on the same sphere, the plane of the first great circle is ... parallel to the plane of the second.

29 A section of an oblique prism made by a plane parallel to a lateral edge of the prism is ... a rectangle.

30 The perimeter of a spherical polygon is ... equal to \( 360^\circ \).
Answer four questions from this part. Show all work unless otherwise directed.

31 Prove either a or b:
   a Two lines perpendicular to the same plane are parallel. \[10\]

   OR

   b In two polar triangles each angle of one has the same measure as the supplement of the side lying opposite it in the other. \[10\]

32 In the accompanying diagram plane \( p \) is perpendicular to plane \( s \), intersecting \( s \) in line \( QR \). Line \( EF \) (in plane \( s \)) is perpendicular to \( QR \) at \( E \). Line \( AB \) is drawn parallel to \( EF \) through any point \( A \) in \( p \) not on \( QR \). Line \( AE \) is drawn. Prove: Angle \( BAE \) is a right angle. \[10\]

33 Given plane \( m \) and point \( P \) at distance \( d \) from \( m \). Describe fully the locus of points
   a at a distance \( f \) from \( P \) \[2\]
   b at a distance \( g \) from \( m \) \[2\]
   c that satisfy both conditions in \( a \) and \( b \) if
     (1) \( f = d \) and \( g = 2d \) \[3\]
     (2) \( f = 2d \) and \( g < d \) \[3\]

34 Each leg of an isosceles triangle is \( 3m \) units in length and the base is \( 2m \) units. A line \( s \) is drawn through the vertex of the triangle parallel to the base. The triangle is revolved through \( 360^\circ \) about line \( s \) with the base of the triangle always remaining parallel to \( s \). Find in terms of \( m \) the volume of the resulting solid. \[10\]

35 The accompanying figure represents one type of prismatoid with altitude equal to \( h \). Line \( EF \) is parallel to side \( AB \) of rectangle \( ABCD \). Let \( EF = p \), \( AB = 2p \) and \( BC = 2q \). Using the prismatoid formula, find the volume of the solid in terms of \( h \), \( p \) and \( q \). \[10\]

*36 Answer either a or b:
   a In spherical triangle \( ABC \), angle \( C = 90^\circ \), side \( b = 28^\circ \) and side \( c = 102^\circ \). Find side \( a \) to the nearest degree. \[10\]

   OR

   b Points \( A \) \((-2, 1, 4)\) and \( B \) \((3, -3, 6)\) are located with reference to three mutually perpendicular axes: \( x \), \( y \) and \( z \).
     (1) Write an equation of the \( xy \)-plane. \[2\]
     (2) Write an equation of the plane through point \( A \) parallel to the \( yz \)-plane. \[2\]
     (3) Find the length of line segment \( AB \). \[3\]
     (4) Find the coordinates of the midpoint of line segment \( AB \). \[3\]

* This question is based on optional topics in the syllabus.
**12B**

**INSTRUCTIONS FOR RATING**

**TWELFTH YEAR MATHEMATICS**

12B (Solid Geometry)

Monday, June 19, 1961 — 1:15 to 4:15 p.m., only

Use only red ink or pencil in rating Regents papers. Do not attempt to correct the pupil’s work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

**Part I**

Allow 2 credits for each correct answer; allow no partial credit. For questions 18-26, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

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[OVER]
Part II

(33) There are many ways of describing these loci. Each description should include shape and position. For instance, phrases such as the following should be allowed credit as indicated:

a. a spherical surface with center at \( P \) and radius \( f \)  [2]

b. two planes parallel to \( m \), one on either side and at a distance \( g \) from \( m \)  [2]

c. (1) the point of tangency between the spherical surface described in a and one of the planes described in b  [3]

(2) two circles, the intersections formed by the spherical surface described in a and the two planes described in b  [3]

(34) \( \frac{32}{3} \pi m^a \)  [10]

(35) \( \frac{5}{3} \) \( hpq \)  [10]

(36) a 104°  [10] [Note: 3 credits should be deducted if 76° is given as the answer.]

b. (1) \( z = 0 \)  [2]

(2) \( x = -2 \) or \( x + 2 = 0 \)  [2]

(3) \( 3\sqrt{5} \)  [3]

(4) \( (\frac{3}{2}, -1, 5) \)  [3]