

SOLID GEOMETRY

Friday, June 14, 1957—1:15 to 4:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. A right section of an oblique prism is a square whose side is 2. The lateral area of the prism is 48. Find a lateral edge of the prism. 1.....
 2. Find the lateral area of the regular triangular pyramid whose base edge is 6 and whose slant height is 8. 2.....
 3. The upper and lower base edges of a frustum of a regular square pyramid are a and $2a$, respectively. The slant height of the frustum is s . Express the lateral area of the frustum in terms of a and s . 3.....
 4. How many square feet of sheet metal are needed to make a 7-foot length of stovepipe in the shape of a circular cylindrical surface if the radius of a cross section is $\frac{1}{3}$ foot? [Use $\pi = \frac{22}{7}$ and make no allowance for waste.] 4.....
 5. Find the volume of the right circular cone whose altitude is 3 and whose elements are inclined 45° to its base. 5.....
 6. Find to the nearest degree the angle that a line segment 5 inches long makes with a plane if the length of its projection on the plane is 2 inches. 6.....
 7. The area of the base of a pyramid is 36 square feet, and the area of the section formed by a plane parallel to the base and 3 feet from the vertex of the pyramid is 16 square feet. Find the number of feet in the altitude of the pyramid. 7.....
 8. Find the diameter of the sphere which is circumscribed about a cube whose edge is 6. 8.....
 9. Find the volume of the sphere whose radius is 2. 9.....
 10. A plane intersects a sphere whose radius is 10 inches at a distance of 7 inches from its center. Find the ratio of the areas of the two zones into which the surface of the sphere is divided by the plane. 10.....
 11. A lune whose angle is 70° lies on a sphere whose area is 36 square inches. Find the number of square inches in the area of the lune. 11.....
- Directions (12-17): Indicate the correct completion for each of the following by writing the letter a , b or c on the line at the right.
12. A solid metal circular cylinder whose radius is 1 inch weighs 2 ounces. The number of inches in the radius of a similar cylinder of the same material that weighs 16 ounces is (a)4 (b)2 (c)8 12.....
 13. The face angles of a trihedral angle may be in the ratio (a)1:2:3 (b)2:2:5 (c)2:3:4 13.....

14. The locus of points on the surface of a sphere and equidistant from two points on the sphere is (a) a small circle (b) a great circle (c) two points 14.....
15. A plane is determined if it passes through a given point and is (a) parallel to a given line (b) perpendicular to a given plane (c) perpendicular to a given line 15.....
16. Two spherical triangles on a sphere are *always* either congruent or symmetrical if the following given parts of one are equal to the corresponding parts of the other: (a) any two sides and an angle (b) any two angles and a side (c) three angles 16.....
17. The perimeter of a convex spherical polygon drawn on a sphere whose radius is 3 may be equal to (a) 4π (b) 6π (c) 8π 17.....

Directions (18-20): For each of the following, tell whether the statement is always true, sometimes true or never true by writing the word always, sometimes or never on the line at the right.

18. If two lines are skew, a plane parallel to one of the lines will be parallel to a plane parallel to the other line. 18.....
19. If two face angles of a trihedral angle are equal, the dihedral angles opposite these angles are equal. 19.....
20. If two planes cut off equal segments on two parallel lines, the planes are parallel. 20.....

Part II

Answer three questions from this part.

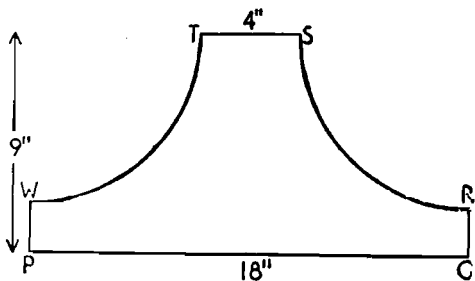
21. Prove: If each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane. [10]
22. In sphere O , AOB is the axis of the great circle RST and PQ is a line tangent to the sphere at A . Prove that PQ is parallel to the plane of the circle RST . [10]
23. Prove: If the first of two spherical triangles is the polar triangle of the second, then the second is the polar triangle of the first. [10]
24. Perpendicular planes R and S intersect in line c , and P is a point in R .
- State in full the locus of points
 - equally distant from R and S [3]
 - at a given distance d from P [2]
 - What is the intersection of the two loci given in answer to part a if P is 6 inches from c and
 - $d = 8$ inches? [2]
 - $d = 3\sqrt{2}$ inches? [3]
25. The base of a pyramid is a rectangle whose dimensions are $3a$ and $4a$. The altitude of the pyramid meets the base at the intersection of the diagonals of the base. A lateral edge of the pyramid makes an angle θ with the base. Show that the volume V of the pyramid is given by the formula $V = 10a^3 \tan \theta$. [10]

Part III

Answer two questions from this part. Show all work.

26. The sum of the sides of a spherical triangle is 310° . If the diameter of the sphere is 9 inches, find, to the nearest tenth of a square inch, the area of the polar triangle. [10]

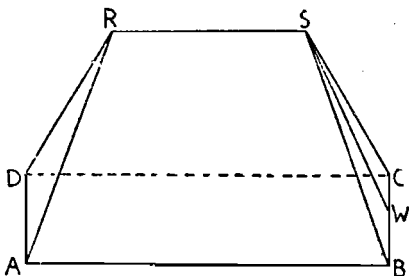
27. The figure at the right represents every right section of a metal casting. Angles P and Q are right angles, TS is parallel to PQ , and WT and RS are quadrants of equal circles. Using the dimensions given on the figure, find, to the nearest ten pounds, the weight of the casting if it is 30 inches long and made of steel weighing 480 pounds per cubic foot. [Use $\pi = 22/7$.] [10]



28. AOB is a sector of circle O , and M is the midpoint of arc AB . Radius OM intersects chord AB at C . Sector AOB is rotated through 180° about OM as an axis to form a solid.

- If angle $AOB = 118^\circ$ and $OA = 10$, find OC and AC to the nearest integer. [4]
- Using the results found in answer to part a , find the total area of the solid. [Answer may be left in terms of π .] [6]

*29. The accompanying figure represents a wedge which is a special type of prismatoid. Base $ABCD$ is a rectangle; faces ARD and BSC are congruent isosceles triangles and RS is a line segment parallel to base $ABCD$; $AB = 30$, $BC = 12$, $RS = 14$ and the altitude SW of triangle $BSC = 17$.



Using the formula

$$V = \frac{h}{6} (B_1 + B_2 + 4m), \text{ find the volume of the wedge. [10]}$$

* This question is based on an optional topic in the syllabus.

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Friday, June 14, 1957—1:15 p.m. to 4:15 p.m., only

Part III

Directions: The following questions are based upon the optional topics of the twelfth year syllabus. *Either one or both* may be substituted for *any one or two* of the questions on part III of the examination in solid geometry.

30. In spherical triangle ABC , $c = 90^\circ$, $a = 102^\circ$ and $b = 144^\circ$.
- Find the angles of the polar triangle of triangle ABC . [3]
 - Find, to the nearest degree, the shortest side of the polar triangle. [7]
31. Given the points $A(4, 0, 0)$, $B(1, 2, 6)$ and $C(0, -5, 0)$.
- Find the coordinates of the midpoint of the line segment AB . [$2\frac{1}{2}$]
 - Write the equation of the plane through B parallel to the xy -plane. [$2\frac{1}{2}$]
 - Write an equation of the plane through A and C parallel to the z -axis. [$2\frac{1}{2}$]
 - Find the distance between A and B . [$2\frac{1}{2}$]

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Wednesday, January 22, 1958—9:15 a.m. to 12:15 p.m., only

Directions: The following questions are based upon the optional topics of the twelfth year syllabus. Question 30 may be substituted for any question in part II *only*. Question 31 may be substituted for any question in part III *only*.

Part II

30. a. Write an equation of the plane parallel to the yz -plane and passing through the point $(2, -3, 5)$. [2]
- Find the distance between the points $(-1, 2, 5)$ and $(6, -2, 10)$. [Answer may be left in radical form.] [3]
 - Write an equation of the plane whose x -, y - and z -intercepts are 3, 4 and -1 , respectively. [3]
 - Write an equation of the sphere whose center is the origin and whose radius is 8. [2]

Part III

31. Given spherical triangle ABC in which angle C equals 90° , side a equals 129° and angle B equals 43° .
- Find side b to the nearest degree. [7]
 - Using the given data, write an equation that could be used to find angle A . [3]