

The University of the State of New York

306TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, June 23, 1949 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that in two polar triangles each angle of one has the same measure as the supplement of the side lying opposite to it in the other. [10]

22 Prove that two lines perpendicular to the same plane are parallel. [10]

23 A sphere is tangent to each face of a dihedral angle. Prove that the plane determined by the radii to the points of tangency is perpendicular to the edge of the dihedral angle. [10]

24 Given line  $s$  and point  $P$  on  $s$ . Describe the locus of points

$a$  at a given distance  $r$  from  $s$  [3]

$b$  at a given distance  $d$  from  $P$  [2]

$c$  that satisfies both conditions given in  $a$  and  $b$  if

(1)  $d$  is greater than  $r$  [3]

(2)  $d$  equals  $r$  [2]

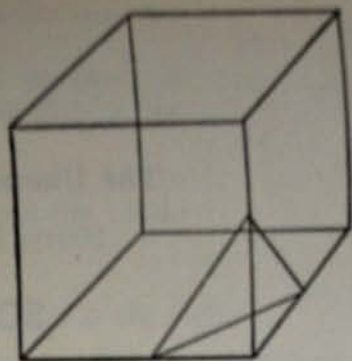
Part III

Answer three questions from part III.

25 A sphere is circumscribed about a cube whose total area is 96. Find the volume of the sphere. [Answer may be left in terms of  $\pi$  and radicals.] [10]

26 Before sharpening, a pencil was in the form of a right circular cylinder 7 inches long and  $\frac{1}{4}$  of an inch in diameter. Sharpening it to a conical point reduced its overall length to  $6\frac{7}{8}$  inches, with the cylindrical part becoming 6 inches long. Find to the nearest hundredth of a cubic inch the volume of the material removed. [Use  $\pi = 3.14$ ] [10]

27 A pyramid is formed by passing a plane through the mid-points of three concurrent edges of a cube as shown in the figure. The edges of the cube are 8 inches long. Find



a the volume of the pyramid to the nearest cubic inch. [4]

b the total area of the pyramid to the nearest square inch. [6]

28 The sum of the face angles at the vertex of a regular pyramid with square base is  $A$ , and the slant height of the pyramid is  $l$ .

a Derive a formula in terms of  $A$  and  $l$  for the lateral area  $S$  of the pyramid. [6]

b Find  $S$  to the nearest square foot if  $A = 240^\circ$  and  $l = 12.3$  feet. [4]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed.

Directions (questions 1-5) — Write on the line at the right of each question the word or number which, when inserted in the blank, will make the statement true.

1 Through a given point outside a given line one plane and only one can be passed ... to the line. 1.....

2 Of two equal line segments oblique to a plane, the one with the ... angle of inclination has the longer projection. 2.....

3 The number of edges of a tetrahedron is equal to the number of ... of a cube. 3.....

4 The area of a sphere is equal to the area of ... great circles of the sphere. 4.....

5 The sum of the angles of a spherical quadrilateral is greater than 360 degrees and less than ... degrees, and may have any value between these limits. 5.....

Directions (questions 6-14) — Write the answer to each question on the line at the right.

6 The radius of the base of a right circular cylinder is 6 and its altitude is 14. Find the total area. [Answer may be left in terms of  $\pi$ .] 6.....

7 One lateral edge of a prism makes an angle of  $60^\circ$  with the base. The area of the base is 40, and the length of a lateral edge is 12. Find the volume of the prism. [Answer may be left in radical form.] 7.....

8 Express the lateral area of a cone of revolution in terms of its altitude  $h$  and the radius  $r$  of its base. 8.....

9 The base edges of a frustum of a regular square pyramid are 8 and 14 and the slant height is 5. Find the altitude. 9.....

10 The radii of the bases of a frustum of a cone of revolution are 8 and 6 and the slant height is 3. Find the lateral area of the frustum. [Answer may be left in terms of  $\pi$ .] 10.....

11 A plane is parallel to the base of a pyramid and bisects the altitude. Find the ratio of the area of the section formed to the area of the base. 11.....

12 Find the number of degrees in the angle of a lune with an area of 80 spherical degrees. 12.....

13 The area of a spherical triangle is  $\frac{1}{12}$  of the area of the sphere on which it is drawn. Find the spherical excess of the triangle. 13.....

14 One angle of a birectangular spherical triangle is  $40^\circ$ . Find the number of degrees in the side opposite this angle. 14.....

Directions (questions 15–20) — In *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

- 15 A line perpendicular to one of two parallel planes is perpendicular to the other plane also. 15.....
- 16 If two different planes  $m$  and  $n$  intersect, the locus of points 3 inches from  $m$  and 2 inches from  $n$  consists of four lines. 16.....
- 17 The sum of the plane angles of the three dihedral angles formed by the lateral faces of a right triangular prism is  $180^\circ$ . 17.....
- 18 The sum of the face angles of a convex polyhedral angle is less than  $360^\circ$ . 18.....
- 19 A plane passed through two diagonally opposite edges of a parallelepiped divides it into two congruent triangular prisms. 19.....
- 20 A plane which is the perpendicular bisector of a radius of a sphere divides the sphere into two zones whose areas are in the ratio 1:3. 20.....