

The University of the State of New York
278TH HIGH SCHOOL EXAMINATION
SOLID GEOMETRY
Thursday, June 20, 1940—9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-10) — Indicate the correct answer to each question by writing on the dotted line at the right the letter a , b or c .

1 Two lines are always parallel if (a) they do not intersect, (b) they are perpendicular to the same plane or (c) they are perpendicular to the same line. 1.....

2 Two planes are always parallel if (a) each contains one of two parallel lines, (b) they are perpendicular to the same plane or (c) one contains two intersecting lines each of which is parallel to the other plane. 2.....

3 Two planes are always perpendicular if (a) a line in one is perpendicular to their intersection, (b) a line in one is perpendicular to a line in the other or (c) one contains a line which is perpendicular to the other. 3.....

4 A line is always perpendicular to a given plane if (a) it is perpendicular to each of two intersecting lines in the given plane, (b) it is perpendicular to a given line in the plane or (c) it lies in a plane which is perpendicular to the given plane. 4.....

5 A diagonal of a parallelepiped always (a) is perpendicular to each of the other diagonals, (b) bisects each of the other diagonals or (c) is equal to each of the other diagonals. 5.....

6 A plane is passed parallel to the base of a pyramid so that the section thus formed is equal to one half the base of the pyramid. If the altitude of the pyramid is h , the distance from the vertex of the pyramid to the section is (a) $\frac{h}{\sqrt{2}}$, (b) $\frac{h}{2}$ or (c) $\frac{h}{4}$. 6.....

7 A sphere can be inscribed in (a) any pyramid, (b) any tetrahedron if and only if it is regular or (c) any triangular pyramid. 7.....

8 The volume of any prism is equal to the product of (a) the perimeter of a right section and a lateral edge, (b) the area of a right section and a lateral edge or (c) the area of a right section and the altitude. 8.....

9 The projection of a circle on a plane is (a) always a circle, (b) always an ellipse or (c) sometimes a straight line segment. 9.....

10 If three angles of a spheric quadrilateral are right angles, the fourth angle is (a) acute, (b) right or (c) obtuse. 10.....

Directions (questions 11-15) — Write on the dotted line at the right the expression which, when inserted in the corresponding blank, will make the statement true.

11 The lateral area of a cylinder of revolution whose altitude is equal to a diameter of the base is exactly ... of its total area. 11.....

12 The lateral area of a frustum of a regular triangular pyramid whose base edges are 6 inches and 8 inches and whose slant height is 10 inches is ... square inches.

12.....

13 If the three sides of a spheric triangle are 60° , 80° and 50° , the spheric excess of its polar triangle is ... degrees.

13.....

14 A lune whose angle is 30° is drawn on a sphere whose radius is 3 inches. The area of the lune is ... square inches. [Answer may be left in terms of π .]

14.....

15 A zone whose area is 12π square inches is drawn on a sphere whose radius is 3 inches. The altitude of the zone is ... inches.

15.....

Directions (questions 16-20) — Indicate whether each statement is true or false by writing the word *true* or *false* on the dotted line at the right.

16 If the legs a and b of a right triangle are unequal, the volume of the cone of revolution formed by revolving the triangle about a as an axis is equal to the cone formed by revolving the triangle about b as an axis.

16.....

17 Every section of a circular cylinder made by a plane passing through an element is a parallelogram.

17.....

18 The locus of points equidistant from two given intersecting planes and at a given distance from a fixed point on their line of intersection consists of two circles.

18.....

19 If four straight lines meet in a point, it is impossible for each of them to be perpendicular to all the others.

19.....

20 If two solids are similar but are not equal, it is possible for the ratio of their areas to equal the ratio of their volumes.

20.....

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See instructions for groups II and III on page 1.

Group II

Answer three questions from this group.

21 Prove that if two lines are parallel, every plane containing one of the lines, and only one, is parallel to the other. [10]

22 Prove that if a pyramid is cut by a plane parallel to its base the section is a polygon similar to the base. [10]

23 Three planes, M , R and S , intersect in the same line l . From an external point P , lines a , b and c are drawn perpendicular to M , R and S respectively. Prove that a , b and c lie in the same plane. [10]

24 A sphere of given radius r is to be constructed tangent to a given plane P with its center on a given line l .

a Show how to locate the center of the sphere. [6]

b In general, how many such spheres will there be? [2]

c State a condition under which the construction would be impossible? [2]

*25 The upper base of a prismatoid is a rectangle 9.0 inches by 6.0 inches and the lower base is a rectangle 13.0 inches by 8.0 inches. The altitude of the prismatoid is 10.0 inches and the longer sides of the upper and lower bases are parallel.

a Find, correct to the nearest cubic inch, the volume of the prismatoid. [The formula for the volume of a prismatoid is $V = \frac{h}{6} (B + B' + 4m)$] [7]

b If the dimensions of the prismatoid remain the same but the shorter side of one base is made parallel to the longer side of the other base, does the volume increase, decrease or remain the same? [3]

*This question is based on one of the optional topics and may be used in either Group II or Group III.

Group III

Answer two questions from this group.

26 A right circular cone is inscribed in a sphere. The slant height of the cone is equal to the diameter of the base. Show that the ratio of the volume of the cone to the volume of the sphere is 9:32. [10]

27 Through a metal casting which has the form of a frustum of a cone of revolution a cylindrical hole 2.0 inches in radius is bored, the axis of the cylinder coinciding with the axis of the frustum. The radius of the upper base of the casting is 3.0 inches, the radius of the lower base is 5.0 inches and the height is 6.0 inches. Find, correct to the nearest cubic inch, the volume of the resulting solid. [Use $\pi = 3.14$] [The formula for the volume of a frustum of a

cone is $V = \frac{\pi h}{3} (r_1^2 + r_2^2 + r_1 r_2)$] [10]

28 The slant height s of a regular square pyramid makes with its projection on the base an angle A .

a Show that the volume V of the pyramid is given by the formula $V = \frac{4}{3} s^3 \sin A \cos^2 A$ [4]

b Find, correct to the nearest cubic inch, the value of V , if $s = 2.40$ inches and $A = 35^\circ$ [6]