

The University of the State of New York

266TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, June 18, 1936 — 9.15 a. m. to 12.15 p. m., only

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Instructions

*Do not open this sheet until the signal is given.*

Group I

*This group is to be done first and the maximum time allowed for it is one and one half hours.*

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

## Group I

Answer all questions in this group. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1-15) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 Through a given line which is parallel or ... to a given plane one and only one plane can be passed perpendicular to the given plane. Ans.....

2 Two planes perpendicular to the same ... are parallel. Ans.....

3 The maximum number of planes determined by four parallel lines is .... Ans.....

4 The length of the projection on a plane of a line segment 3 inches long is 2 inches. The line segment is inclined to the plane at an angle of ... degrees. [Express your answer correct to the nearest degree.] Ans.....

5 A right prism and a regular pyramid are constructed on the same base. If the lateral area of the prism is three times that of the pyramid, the height of the prism is to the slant height of the pyramid as 3 is to .... Ans.....

6 A prism and a pyramid have the same altitude. If the base of the prism is twice that of the pyramid, the volume of the prism is exactly ... times that of the pyramid. Ans.....

7 If a right circular cylinder is inscribed in a cube whose edge is 2 inches, the lateral area of the cylinder is ... square inches. [Answer may be left in terms of  $\pi$ .] Ans.....

8 The lateral area of a right circular cone is  $6\pi$  square feet. If the slant height of the cone is 3 feet, the radius of the base is ... feet. Ans.....

9 An element of a right circular cone makes with the base an angle of  $45^\circ$ . The volume  $V$  of the cone, expressed as a function of the radius  $r$  of the base, is  $V = \dots$  Ans.....

10 If the volume of a sphere is  $\frac{500\pi}{3}$  cubic inches, the radius of the sphere is ... inches. Ans.....

11 Two zones are drawn on the same sphere. If the height of the first is twice the height of the second, the area of the first is exactly ... times the area of the second. Ans.....

12 The area of a lune is  $48\pi$  square inches. If the angle of the lune is  $30^\circ$ , the radius of the sphere on which it is drawn is ... inches. Ans.....

13 If the corresponding edges of two similar rectangular solids are in the ratio of  $a:b$ , the ratio of the total areas of the solids is .... Ans.....



14 If one angle of a spheric triangle is  $80^\circ$ , the length of the side opposite this angle in its polar triangle is ... degrees.

Ans.....

15 If the angles of a spheric quadrilateral are  $115^\circ$ ,  $105^\circ$ ,  $140^\circ$  and  $135^\circ$ , the spheric excess of the quadrilateral is ... degrees.

Ans.....

Directions (questions 16-20) — Indicate the correct answer to each of the following questions by writing on the dotted line at the right the letter (a), (b) or (c):

16 The locus of the center of a sphere of given radius  $r$  and tangent to a given line  $l$  is (a) a line parallel to  $l$  and  $r$  distance from it, (b) a plane parallel to  $l$  and  $r$  distance from it or (c) a cylindric surface whose axis is  $l$  and whose radius is  $r$ .

Ans.....

17 If a line is perpendicular to a plane, all the perpendiculars from the given line to another line in the plane not passing through the foot of the perpendicular are (a) coincident, (b) concurrent or (c) parallel.

Ans.....

18 If a plane is passed parallel to the base of a pyramid and bisects the altitude, then the volume of the pyramid cut off and the volume of the given pyramid are in the ratio (a) 1:7, (b) 1:4 or (c) 1:8

Ans.....

19 The face angles of a trihedral angle may be (a)  $50^\circ$ ,  $80^\circ$ ,  $50^\circ$ , (b)  $170^\circ$ ,  $110^\circ$ ,  $90^\circ$  or (c)  $50^\circ$ ,  $20^\circ$ ,  $90^\circ$

Ans.....

20 In the spheric triangle  $ABC$ ,  $A$  is the pole of the arc  $BC$ . If there are  $60^\circ$  in the arc  $BC$ , the triangle  $ABC$  is (a) equiangular, (b) its own polar triangle or (c) birectangular.

Ans.....

See instructions for groups II and III on page 1.

## Group II

Answer three questions from this group.

21 Prove that all the perpendiculars that can be drawn to a given line at a given point lie in the plane perpendicular to the line at the given point. [10]

22 Prove that if the first of two spheric triangles is the polar triangle of the second, then the second is the polar triangle of the first. [10]

23 Prove that if from any point in a dihedral angle perpendiculars are drawn to the faces, the angle between the perpendiculars is the supplement of the plane angle of the dihedral angle. [10]

24 In a right triangle  $ABC$  with  $AB$  the hypotenuse,  $CA$  is 6 inches and  $CB$  is 8 inches.

a Describe completely the locus of points equidistant from  $C$  and  $A$ . [3]

b Describe completely the locus of points equidistant from  $A$ ,  $B$  and  $C$ . [3]

c How far from the hypotenuse is a point that is 13 inches from each of the three vertices of the triangle? [4]

\*25 Using Cavalieri's Theorem, prove that any two pyramids having equal bases and equal altitudes are equal. [10]

## Group III

Answer two questions from this group.

26 When an irregular piece of metal is completely immersed in water in a circular cylindrical container, the level of the water rises  $4\frac{1}{4}$  inches. The radius of the base of the cylinder is  $3\frac{1}{4}$  inches. Find, correct to the nearest cubic inch, the volume of the piece of metal. [Use  $\pi = 3.142$ ] [10]

27 In a frustum of a regular square pyramid, the edge of the lower base is  $a$  and the edge of the upper base is  $b$ . The slant height makes with the lower base an angle  $x$ . Show that the lateral area  $S$  of the frustum is given by the formula

$$S = \frac{a^2 - b^2}{\cos x} \quad [10]$$

28 Two angles of a spheric triangle are  $75^\circ$  and  $80^\circ$  and the radius of the sphere on which the triangle is drawn is 3 inches.

a If the area of the triangle is 11 square inches, find the third angle of the triangle. [Use  $\pi = 3.2$ ] [8]

b If the radius of the sphere is multiplied by 2, by what number is the area of the spheric triangle multiplied? [2]

\* This question is based on one of the optional topics in the syllabus.