The University of the State of New York

236TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Friday, June 18, 1926 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Name the author of the textbook you have used in your study of solid geometry.

Answer eight questions, including not more than four from group I.

Group I

Do not answer more than four questions from this group.

1 Prove that if each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane. [$12\frac{1}{2}$]

2 Prove that if two intersecting straight lines are parallel respectively to two other intersecting straight lines, the plane of the first pair is parallel to the plane of the second pair or coincides with it. [$12\frac{1}{2}$]

3 Prove that the lateral area of a frustum of a regular pyramid is equal to the product of the slant height and half the sum of the perimeters of the two bases. [$12\frac{1}{2}$]

4 Prove that every section of a sphere made by a plane is a circle. [$12\frac{1}{2}$]

5 Prove that the sum of the angles of a spheric triangle is greater than two, and less than six, right angles. [$12\frac{1}{2}$]

Group II

Irrational results may be left in the form of π and radicals unless otherwise stated.

6 Given two points, A and B, 8″ apart; find, if possible, the locus of points (*a*) equidistant from A and B and 5″ from A, (*b*) equidistant from A and B and 4″ from A, (*c*) equidistant from A and B and 3″ from A. [6, $3\frac{1}{2}$, 3]

7 The lateral areas of two similar cones of revolution are 625 and 169 respectively. If the altitude of the first cone is 5, find (*a*) the altitude of the second cone, (*b*) the ratio of the volumes of the two cones. [$12\frac{1}{2}$]

8 In a regular quadrangular pyramid a lateral edge is equal to a side of the base; find the total area and the volume in terms of the side of the base. [$12\frac{1}{2}$]

9 Given the triangle ABC; AD and BE are two parallel lines oblique to the plane ABC. The plane through AC and AD and the plane through BC and BE intersect in CF. Prove that CF is parallel to BE. [$12\frac{1}{2}$]

10 The sides of a spheric triangle are 70°, 80° and 120° and the area of the polar triangle is 18π square feet; on the same sphere find the area of a zone whose altitude is half the radius. [$12\frac{1}{2}$]

11 The distance from the center of a sphere to the plane of a small circle is half the radius; two cones are constructed with this circle as a common base and with their vertices at the extremities of the diameter perpendicular to the plane of the circle. Find (*a*) the ratio of the lateral area of the larger cone to the lateral area of the sphere, (*b*) the ratio of the volume of the larger cone to the volume of the sphere. [$8\frac{1}{2}$, 4]

12 Given right triangle ABC with O the mid-point of hypotenuse AB and with OC drawn; P is a point not in the plane ABC, such that $PA = PB = PC$. Prove that (*a*) triangles POA and POC are congruent, (*b*) PO is perpendicular to the plane ABC. [4, $8\frac{1}{2}$]