The University of the State of New York

231st High School Examination

SOLID GEOMETRY

Monday, June 16, 1924 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Name the author of the textbook you have used in your study of solid geometry.

Answer eight questions, including not more than four from group I.

Group I

Do not answer more than four questions from this group.

1 Prove that all the perpendiculars that can be drawn to a straight line at a given point lie in the plane perpendicular to the line at the point. [$12\frac{1}{2}$]

2 Prove that if each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane. [$12\frac{1}{2}$]

3 Prove that the plane passing through two diagonally opposite edges of a parallelepiped divides the parallelepiped into two equal (equivalent) triangular prisms. [$12\frac{1}{2}$]

4 Prove that if a pyramid is cut by a plane parallel to its base, the section is a polygon similar to the base. [$12\frac{1}{2}$]

5 Prove that if the first of two spheric triangles is the polar triangle of the second, then reciprocally, the second is the polar of the first. [$12\frac{1}{2}$]

Group II

Irrational results may be left in the form of π and radicals unless otherwise stated.

6 Find the locus of points in space at a given distance from a given line and also equidistant from two given points on that line. [$12\frac{1}{2}$]

7 The volumes of two similar cones of revolution are 343 and 512 respectively. If the lateral area of the first cone is 196, what is the lateral area of the second? [$12\frac{1}{2}$]

8 Find the lateral area and the volume of a frustum of a regular quadrangular pyramid, the sides of whose bases are 17 and 7 respectively and whose altitude is 12. [$12\frac{1}{2}$]

9 A right circular cylinder is circumscribed about a sphere.

a Find the ratio of the total surfaces of the two solids. [6]

b Find the ratio of the volumes of the two solids. [$6\frac{1}{2}$]

10 A spheric triangle on the earth's surface (regarded as a sphere) has one of its vertices at the north pole and the other two vertices on the equator. Find the size of each of the angles if the area of the triangle is equal to one half the area of a zone on the earth's surface whose altitude is equal to one half the radius. [$12\frac{1}{2}$]

11 A–BCD is a tetrahedron and E, F, G, H are the mid points of the edges AD, AC, BC, BD respectively. Prove that $EFGH$ is a parallelogram. [$12\frac{1}{2}$]

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