The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Wednesday, January 22, 1958 — 9:15 a.m. to 12:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 A rectangular parallelepiped has the dimensions 6, 6 and 7. Find the length of a diagonal of the parallelepiped.

1.....

2 Find the volume of a cone of revolution whose radius is 6 and whose altitude is 12.

2.....

3 The area of the base of a pyramid is 112 and the area of a section parallel to the base is 63. If the altitude of the pyramid is 12, find the distance from the section to the vertex.

3.....

4 An element of a right circular cone makes an angle of 60° with the base. If the radius of the cone is r, find the lateral area in terms of r.

4.....

5 The base edges of a frustum of a regular triangular pyramid are 2 and 6. If the lateral area is 60, find the slant height.

5.....

6 A line AB makes an angle of 59° with a plane. If AB is 7.2, find, to the *nearest tenth*, the projection of AB on the plane.

6.....

7 A lune and a spherical triangle on the same sphere have equal areas. If the angle of the lune is 10°, find the number of degrees in the sum of the angles of the triangle.

7.....

8 The total area of a regular octahedron is $72\sqrt{3}$. Find an edge of the octahedron.

8.....

[OVER]

9 The area of a great circle of a sphere is k. Express the area of the sphere in terms of k. 9.....

10 The perimeter of an equilateral spherical triangle is 210°. Find the number of degrees in the sum of the angles of the polar triangle. 10.....

11 Find the number of planes that are determined by the end points of two skew line segments. 11.....

12 Find the number of cubic inches in the volume of a right circular cylinder 14 inches long and 2 inches in radius. [Use the approximation $\pi = \frac{22}{7}$.] 12.....

Directions (13–16): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b*, *c* or *d*.

13 Two face angles of a trihedral angle are 87° and 103°. The third face angle may be (*a*) 16° (*b*) 160° (*c*) 170° (*d*) 190° 13.....

14 A plane is passed through a sphere such that its intersection with the sphere forms a circle whose distance from its pole is 60°. The sphere is divided into two zones whose areas are in the ratio of (*a*) 2:5 (*b*) 1:2 (*c*) 1:3 (*d*) 1:4 14.....

15 The perimeter of a trirectangular spherical triangle is 12π. The radius of the sphere is (*a*) 5 (*b*) 6 (*c*) 7 (*d*) 8 15.....

16 The locus of points that are equally distant from two parallel planes and also equally distant from two intersecting lines on one of the planes is (*a*)two points (*b*)a line (*c*)two parallel lines (*d*)two perpendicular lines 16.....

Directions (17–20): For *each* of the following, tell whether the statement is always true, sometimes true or never true by writing the word *always, sometimes* or *never* on the line at the right.

17 A straight line segment joining two points on a sphere lies entirely on the surface of the sphere. 17.....

18 If two intersecting planes are each tangent to a cylindrical surface, their line of intersection is parallel to an element. 18.....

19 Two spheres tangent to the same line at the same point are tangent to each other. 19.....

20 Pyramids with equal bases and equal altitudes are congruent. 20.....

Part II

Answer three questions from this part.

21 Prove: If a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to the plane of the two lines. [10]

22 Prove: The sum of the angles of a spherical triangle is greater than 180° and less than 540°. [10]

23 Line segments AB and CD are both equal and parallel. They are oblique to plane M and meet it at points A and C. Parallel lines are drawn from B and D to meet plane M at points R and S, respectively. Prove: $AR = CS$. [10]

24 Given plane M, line k in plane M and point P on line k.

(A) Describe fully the locus of points

(1) a given distance d from M [2]

(2) a given distance s from k [2]

(3) a given distance r from P [2]

(B) Indicate the correct completion for *each* of the following by writing the letter a, b, c or d after the numbers 1 and 2 on your answer paper.

(1) The locus of points satisfying both (1) and (2) of A if $s = d$ is (a)one line (b)two lines (c)three lines (d)four lines [2]

(2) The locus of points satisfying both (2) and (3) of A if $s = r$ is (a)one point (b)one circle (c)two circles (d)one line [2]

25 The altitude of a regular square pyramid is h and each lateral face of the pyramid makes an angle θ with the base.

a Show that the volume of the pyramid is $\dfrac{4h^3}{3\tan^2\theta}$. [5]

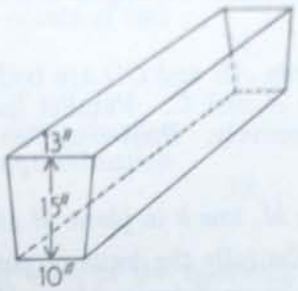
b Show that the lateral area of the pyramid is $\dfrac{4h^2}{\sin\theta\tan\theta}$. [5]

Part III

Answer two questions from this part. Show all work.

26 An equilateral spherical triangle is equal in area to a zone on the same sphere. If the altitude of the zone is 2 and the radius of the sphere is 14, find, to the *nearest degree*, one angle of the triangle. [10]

27 An open trough has the form of a right prism with isosceles trapezoids as ends, as shown in the figure at the right. The bases of the trapezoids are 10 inches and 13 inches and the altitude is 15 inches. The length of the trough is 6 feet. Find, to the *nearest gallon*, the number of gallons in the trough when it is filled to a depth of 10 inches. [1 gal. = 231 cu. in.] [10]

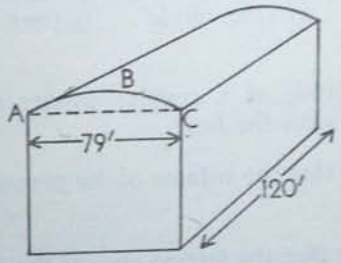


28 A regular square prism whose altitude is twice a base edge is inscribed in a sphere. A base edge of the prism is x.

a Show that the volume of the sphere is equal to $\pi x^3\sqrt{6}$. [4]

b If the volume of the sphere is 233, find the value of x to the *nearest tenth*. [Use the approximation $\pi = 3.14$.] [6]

29 The figure at the right shows a building whose roof forms part of a right circular cylindrical surface. Arc $ABC = 60°$, the distance from A to C is 79 feet and the building is 120 feet long. If the roof costs \$2.72 per square yard, find the cost of the roof to the *nearest ten dollars*. [10]



[4]

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
SOLID GEOMETRY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For questions 13–16, allow credit if the pupil has written the correct answer instead of the letter *a*, *b*, *c* or *d*.

(1) 11

(2) 144π *or* 452.2

(3) 9

(4) $2\pi r^2$

(5) 5

(6) 3.7

(7) 200

(8) 6

(9) $4k$

(10) 330

(11) four

(12) 176

(13) *b*

(14) *c*

(15) *d*

(16) *d*

(17) never

(18) always

(19) sometimes

(20) sometimes

157