SOLID GEOMETRY

Wednesday, January 23, 1957—9:15 a.m. to 12:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive **2½** *credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of* π *or in radical form.*

1. Find a diagonal of a rectangular solid whose edges are **3**, **4** and 12. 1.....

2. Find the lateral surface of the cylinder of revolution formed when a rectangle, 3 by 5, is rotated through 360° about the side 5. 2.....

3. Find the total surface of a regular tetrahedron whose edge is 4. 3.....

4. Find the lateral area of a regular square pyramid whose base edge is 12 and whose altitude is 8. 4.....

5. A zone drawn on a sphere whose radius is 6 has an altitude of 2. Find the area of the zone. 5.....

6. A cone and a cylinder have the same base and equal altitudes. Find the ratio of the volume of the cone to the volume of the cylinder. 6.....

7. The perimeters of the bases of a frustum of a regular pyramid are 14 and 16. The slant height is 4. Find the lateral area of the frustum. 7.....

8. The slant height, s, of a cone of revolution makes an angle of 60° with its projection on the base of the cone. Find in terms of s the lateral area of the cone. 8.....

9. A lune drawn on a sphere of radius 6 inches has an angle of 10°. Find the number of square inches in the area of the lune. 9.....

10. Two cylinders of revolution have equal altitudes, and the radius of the base of the first is twice the radius of the base of the second. Find the ratio of the volume of the first cylinder to the volume of the second. 10.....

11. The area of the base of a pyramid is 36 square inches. Find the number of square inches in the area of the section of the pyramid made by a plane that is parallel to the base and that intersects the altitude one-third of the distance from the vertex to the base. 11.....

Directions (12-15): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

12. If the radius of a hemisphere is multiplied by 2, the volume is multiplied by (*a*) 2 (*b*) 4 (*c*) 8 12.....

13. A line that is perpendicular to the first of two perpendicular planes and that does *not* lie in either plane is (*a*) perpendicular to the second (*b*) parallel to the second (*c*) oblique to the second 13.....

14. If the lateral faces of a regular pyramid are all equilateral triangles, the base of the pyramid *may* be a (*a*) square (*b*) regular hexagon (*c*) regular octagon 14.....

15. The locus of points equidistant from two intersecting planes consists of (*a*) one plane (*b*) two planes (*c*) one line 15.....

Directions (16-20): For *each* of the following tell whether the statement is always true, sometimes true or never true by writing the word ***always***, ***sometimes*** or ***never*** on the line at the right.

16. Two small circles on a sphere intersect. 16.....

17. Two isosceles spherical triangles on the same sphere are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other. 17.....

18. If two planes are parallel to the same line, they are parallel to each other. 18.....

19. If a plane intersects each of two other intersecting planes, but not in their line of intersection, the three lines of intersection are concurrent. 19.....

20. A straight line oblique to a plane is oblique to every line in the plane through the point of intersection of the given line and the plane. 20.....

Part II

Answer three questions from this part.

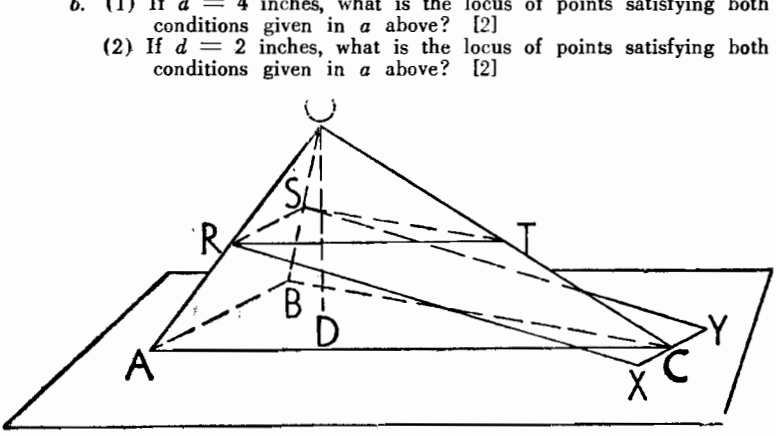
21. Prove: If two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other. [10]

22. Prove: In two polar triangles each angle of one has the same measure as the supplement of the side lying opposite it in the other. [10]

23. Given a sphere whose radius is 4 inches and which is tangent to a plane.

a. Describe fully the locus of points
(1) 2 inches from the sphere [3]
(2) at a given distance d inches from the plane [3]

b. (1) If $d = 4$ inches, what is the locus of points satisfying both conditions given in *a* above? [2]
(2) If $d = 2$ inches, what is the locus of points satisfying both conditions given in *a* above? [2]



24. In pyramid *O-ABC* above, a plane perpendicular to the altitude *OD* intersects face *OAB* in line *RS*.

Prove that the plane determined by *RS* and *C* intersects the plane of the base of the pyramid in a line *XY*, which is parallel to *AB*. [10]

25. A cylinder of revolution is circumscribed about a sphere whose radius is *r*.
   a. Find the ratio of the surface of the sphere to the total area of the cylinder. [5]
   b. Find the ratio of the volume of the sphere to the volume of the cylinder. [5]

Part III

Answer two questions from this part. Show all work.

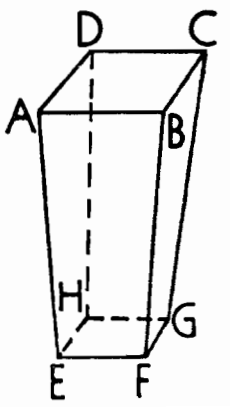
26. The altitude of a regular hexagonal pyramid is 13 and each face makes an angle of 55° with the base. Find to the *nearest integer*
   a. the apothem of the base [3]
   b. the edge of the base [4]
   c. the volume of the pyramid [3]

27. *a.* If the area of a zone on a sphere is 84π and its altitude is 7, find the radius of the sphere. [3]
   b. Find the area of a spherical triangle on the same sphere if the sides of its polar triangle are 60°, 85° and 95°. [Use $\pi = 22/7$.] [7]

28. An oil reservoir has the shape of a frustum of a right circular cone 21 feet deep. The radius of the lower base is 28 feet and the radius of the upper base is 40 feet. The reservoir is filled with oil to a depth of 14 feet. Find, to the *nearest thousand gallons*, the number of gallons of oil that must be added to fill the whole reservoir.
[$V = 1/3\pi h(r_1^2 + r_2^2 + r_1r_2)$. 1 cu. ft. = 7½ gal. Use $\pi = 22/7$.] [10]

*29. A storage bin in the shape of a prismatoid is shown at the right. The planes of rectangles *ABCD* and *EFGH* are parallel and 40 inches apart; $AB = 20$ inches, $BC = 18$ inches, $EF = 14$ inches and $FG = 10$ inches. Find the volume of the bin to the *nearest tenth of a cubic foot.* [The formula for the volume of a prismatoid is

$$V = \frac{h}{6}(B + B' + 4m)\text{]}. \quad [10]$$



*This question is based on an optional topic in the syllabus.

TWELFTH YEAR MATHEMATICS

12B

12B (Solid Geometry)

Thursday, January 26, 1956—9:15 a.m. to 12:15 p.m., only

Part III

Directions: The following questions are based upon the optional topics of the twelfth year syllabus. *Either one or both* may be substituted for *any one or two* of the questions on part III of the examination in solid geometry.

29. Given spherical triangle ABC in which angle $C = 90°$, angle $A = 64°\ 20'$ and side $b = 60°$. Find side c to the *nearest degree.* [10]

30. *a.* Find the coordinates of the mid-point of the line segment whose end points are (1, 2, 7) and (3, —2, 7). [3]

    b. Find the x, y and z intercepts of the plane whose equation is $2x - 3y + 8z = 24$. [1, 1, 1]

    c. Write an equation of the plane parallel to the z-axis and passing through the points (4, 0, 0) and (0, —3, 0). [2]

    d. Write the equation of the plane parallel to the xy-plane and passing through the point (5, —1, 7). [2]

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Wednesday, January 23, 1957—9:15 a.m. to 12:15 p.m., only

Part III

Directions: The following questions are based upon the optional topics of the twelfth year syllabus. *Either one or both* may be substituted for *any one or two* of the questions on part III of the examination in solid geometry.

30. *a.* Write an equation of the plane parallel to the xz-plane and passing through the point (2, 3, —6). [2]

    b. Find the distance between the points (2, 3, —6) and (—5, 6, 1). [3]

    c. Find the coordinates of the midpoint of the line segment joining the points (5, 6, 2) and (—2, 4, —6). [2]

    d. Write an equation of the plane whose x-, y- and z-intercepts are 5, —5 and 4, respectively. [3].

31. Given spherical triangle ABC in which angle $C = 90°$, side $c = 62°$ and side $b = 37°$.

    a. Find side a to the *nearest degree.* [7]

    b. Using the given data, write an equation that could be used to find angle A. [3]