

The University of the State of New York

317TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 22, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

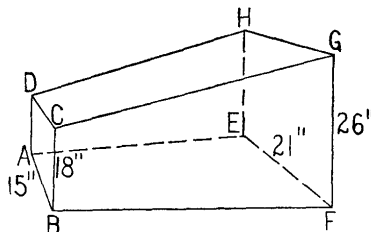
21 Prove that if two angles, not in the same plane, have their sides respectively parallel and extending in the same direction from their vertices, they are equal. [10]

22 Prove that if a line is perpendicular to one of two intersecting planes which are oblique to each other, its projection on the other plane is perpendicular to the line of intersection of the two planes. [10]

23 Prove that if a point on a sphere is at a quadrant's distance from each of two other points on the sphere, not the extremities of a diameter, it is the pole of the great circle through these points. [10]

*24 The accompanying figure represents a coal chute. $ABCD$ and $EFGH$ are rectangles whose planes are parallel and 4 feet apart. Using the prismatoid formula, $V = \frac{h}{6} (B + B' + 4m)$,

find to the nearest cubic foot the volume of the chute if $AB = 15$ inches, $BC = 18$ inches, $EF = 21$ inches and $FG = 26$ inches. [10]



* This question is based upon one of the optional topics in the syllabus.

Part III

Answer three questions from part III.

25 A zone of one base and an equilateral spherical triangle are drawn on a sphere whose radius is 15. The area of the triangle is equal to the area of the zone. If the radius of the base of the zone is 12, find

a the altitude of the zone [3]

b the area of the zone [Answer may be left in terms of π .] [2]

c each angle of the triangle [5]

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26 A cylindrical bar of metal whose altitude is 6 inches and whose radius is 4 inches is melted and recast into a sphere. Find to the *nearest tenth of an inch*, the radius of the sphere. [10]

27 A container has the shape of a frustum of a regular square pyramid. The height of the frustum is 6 in. and the edges of the upper and lower bases are 10 in. and 4 in. respectively. Water is poured into the container to a depth of 3 in.

a Allowing 231 cu. in. to the gallon, show that the capacity of the container is approximately $1\frac{1}{3}$ gallons. [4]

b Show that the container is approximately .3 full of water.

$$[V = \frac{h}{3} (B + B' + \sqrt{BB'})] \quad [6]$$

28 $ABCD$ is a trapezoid in which angles A and B are right angles and the longer base BC is equal to DC . If BC is represented by s and angle C is represented by θ , show that the total area of the solid formed by revolving the trapezoid through 360° about AD as an axis, is equal to $\pi s^2 \sin \theta (\sin \theta + 3)$. [10]

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Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed.

1 The base of a pyramid is a right triangle whose legs are 6 and 8. If the altitude of the pyramid is 10, find the volume of the pyramid. 1.....

2 Find an edge of a regular tetrahedron whose area is $9\sqrt{3}$. 2.....

3 A rectangular parallelepiped whose dimensions are 6, 3 and 2 is inscribed in a sphere. Find the diameter of the sphere. 3.....

4 Parallel planes R , S and Q are cut by two lines in points A , B , C and D , E , F respectively. If $AB = 6$, $BC = 8$ and $DF = 21$, find EF . 4.....

5 Line segment AB is b inches long and makes an angle x with plane Q . Express the length of the projection of AB on Q in terms of b and x . 5.....

6 A concrete roller is in the form of a right circular cylinder whose length is 7 feet and whose radius is 1 foot. Find the number of square feet it can cover in making 100 revolutions. [Use $\pi = \frac{22}{7}$] 6.....

7 The radii of the bases of a frustum of a cone of revolution are 5 and 9 and the slant height is 10. Find the lateral area of the frustum. [Answer may be left in terms of π .] 7.....

8 A pyramid 15 feet high has a square base 9 feet on a side. Find the area of the section made by a plane parallel to the base and 10 feet from the base. 8.....

9 If a 3-inch orange weighs 9 ounces, how many ounces does a 4-inch orange of the same variety weigh? 9.....

10 If the angle of a lune is 40° , find the number of spherical degrees in its area. 10.....

11 One angle of a spherical triangle is 95° . Find the number of degrees in the side opposite it in the polar triangle. 11.....

12 The areas of the bases of two similar prisms are in the ratio 4 to 9. Find the ratio of their altitudes. 12.....

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Directions (13–15) — Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

13 Two face angles of a trihedral angle are 110° and 130° . The third face angle may be (a) 20° (b) 100° (c) 120° 13.....

14 The number of points which are equidistant from the vertices of a triangle and also 6 inches from the plane of the triangle is (a)1 (b)2 (c)unlimited 14.....

15 If a right circular cylinder whose lateral area is S is circumscribed about a sphere whose area is S' , then (a) $S > S'$ (b) $S = S'$ (c) $S < S'$ 15.....

Directions (16–20) — In *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

16 Through one of two given skew lines, one plane and only one can be passed parallel to the other line. 16.....

17 The angle formed by two great circles on a sphere is equal to the plane angle of the dihedral angle formed by the two planes of the great circles. 17.....

18 If two adjacent lateral faces of a prism are rectangles, the prism is a right prism. 18.....

19 If the projections of two lines on a plane are parallel, the lines are parallel. 19.....

20 The volume of a regular prism is equal to one half the product of its lateral area and the apothem of its base. 20.....