

The University of the State of New York

311TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, January 25, 1951 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that if a line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane. [10]

22 Two lines a and b are parallel. Prove that if a plane is passed through each line so that the planes intersect in a third line, the intersection is parallel to lines a and b . [10]

23 Prove that a spherical angle is measured by the arc of the great circle, described from its vertex as a pole and included between its sides, produced if necessary. [10]

24 Given plane M and point P , 5 inches from M . Describe fully the locus of points

(a) d distance from M [3]

(b) r distance from P [2]

(c) satisfying the conditions in both a and b if

(1) $d = 1$ inch and $r = 7$ inches [3]

(2) $d = 1$ inch and $r = 6$ inches [2]

Part III

Answer three questions from part III.

25 The area of a spherical triangle on a sphere of radius 7 inches is $145\frac{1}{2}$ square inches. Two angles of the triangle are 118° and 100° . Find the third angle of the triangle. [Use $\pi = \frac{22}{7}$] [10]

26 If ice cream is bought by the gallon, find the least number of gallons that must be purchased in order to serve 350 persons if each serving is in the form of a hemisphere whose diameter is 2.1 inches. [1 gallon = 231 cu. in.; use $\pi = \frac{22}{7}$] [10]

[1]

[OVER]

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27 The base of a right prism whose altitude is c is a trapezoid. The parallel sides of the trapezoid are a and b ($a > b$). Two angles of the trapezoid are right angles, and a third angle is the acute angle x .

a Show that the volume V of the prism is given by the formula

$$V = \frac{c(a^2 - b^2) \tan x}{2} \quad [7]$$

b If $a = 12$ inches, $b = 8$ inches, $c = 10$ inches and angle $x = 42^\circ$, find V to the nearest cubic inch. [3]

28 A marble pedestal consists of a frustum of a right circular cone surmounted by a right circular cylinder whose base coincides with the smaller base of the frustum. The radii of the bases of the frustum are 5 inches and 10 inches, and its slant height is 13 inches. The altitude of the cylinder is 3 feet.

a Find the volume of the pedestal. [Answer may be left in terms of π ; the formula for the volume of the frustum is $V = \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2)$] [6]

b If the marble weighs 180 pounds per cubic foot, find the weight of the pedestal to the nearest pound. [Use $\pi = 3.14$] [4]

SOLID GEOMETRY

Fill in the following lines:

Name of pupilName of school

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

- 1 The area of the base of a pyramid is 30 square feet and its volume is 120 cubic feet. Find its altitude. 1.....
- 2 Find the lateral area of a cone of revolution if the radius of its base is 4 feet and its slant height is 7 feet. [Use $\pi = \frac{22}{7}$] 2.....
- 3 A lateral edge of a prism is 8 inches and is inclined 57° to the base. Find the altitude of the prism to the *nearest tenth of an inch*. 3.....
- 4 Express in terms of r the total area of a right circular cylinder whose radius is r and whose altitude is $3r$. [Answer may be left in terms of π .] 4.....
- 5 A pyramid with a square base 15 feet on a side is 60 feet high. Find the area of the section parallel to the base and 20 feet from the base. 5.....
- 6 The total area of a regular tetrahedron is $36\sqrt{3}$. Find an edge of the tetrahedron. 6.....
- 7 Find the lateral area of a frustum of a regular square pyramid whose base edges are 6 and 8 inches and whose slant height is 10 inches. 7.....
- 8 Two tanks in the form of right circular cylinders have their corresponding dimensions in the ratio 3:4. If 9 gallons of paint were needed to paint the smaller tank, how many gallons would be needed to paint the larger tank? 8.....
- 9 Find the polar distance of a point on a great circle of a sphere whose radius is 14 inches. [Use $\pi = \frac{22}{7}$] 9.....
- 10 The area of a great circle of a sphere is 3π . Find the area of the sphere. [Answer may be left in terms of π .] 10.....
- 11 A plane is passed 3 inches from the center of a sphere whose radius is 10 inches. Find the ratio of the areas of the zones thus formed. 11.....
- 12 Find in spherical degrees the area of the lune included between the meridians 20° west and 60° west. 12.....

Directions (questions 13–15) — Indicate the correct answer to *each* question by writing on the line at the right the letter a , b or c .

- 13 An exterior angle of a spherical triangle is (a) greater than (b) less than (c) equal to the sum of the two nonadjacent interior angles. 13.....
- 14 Two face angles of a trihedral angle are 100° and 120° . The third face angle may be (a) 20° (b) 110° (c) 150° 14.....
- 15 The locus of points equidistant from the vertices of a triangular pyramid is (a) a line (b) a point (c) a plane 15.....

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Directions (questions 16–20) — In *each* of the following, if the statement is *always* true, write *true* on the line at the right; if it is *not always* true, write *false*.

- 16 Through one of two skew lines only one plane can be passed parallel to the other. 16.....
- 17 If line a is parallel to plane Q and perpendicular to plane R , then Q is perpendicular to R . 17.....
- 18 If the intersections of two planes by a third plane are parallel, the planes are parallel. 18.....
- 19 A right section of a square prism is a square. 19.....
- 20 If two spherical triangles on the same sphere are mutually equiangular, they are congruent. 20.....