The University of the State of New York

238TH HIGH SCHOOL EXAMINATION

# SOLID GEOMETRY

Friday, January 21, 1927 — 9.15 a. m. to 12.15 p. m., only

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**Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry.**

**The minimum time requirement is five recitations a week for half a school year, or the equivalent.**

**Name the author of the textbook you have used in your study of solid geometry.**

*Answer eight questions, including not more than four from group I.*

## Group I

**Do not answer more than four questions from this group.**

1 Prove that two straight lines perpendicular to the same plane are parallel. [$12\frac{1}{2}$]

2 Prove that the locus of points equally distant from the extremities of a straight line is the plane perpendicular to that line at its middle point. [$12\frac{1}{2}$]

3 Prove that every section of a circular cone made by a plane parallel to its base is a circle the center of which is the intersection of the plane with the axis. [$12\frac{1}{2}$]

4 Prove that the volume of a triangular pyramid is equal to one third the product of its base and its altitude. [$12\frac{1}{2}$]

5 Prove that two symmetric spheric triangles are equal. [$12\frac{1}{2}$]

## Group II

***Irrational results may be left in the form of π and radicals unless otherwise stated.***

6 What is the locus of a point equidistant from two intersecting planes and also at a given distance from the line of intersection of the planes? [$12\frac{1}{2}$]

7 $O$-$ABC$ is a triangular pyramid with vertex $O$ and base $ABC$. $D$ is the mid-point of $OB$ and $E$ is the mid-point of $OC$. Prove that the plane determined by $A$, $D$ and $E$ cuts the plane of the base $ABC$ in a line parallel to $BC$. [$12\frac{1}{2}$]



8 The bases of two regular pyramids are equal squares with sides $a$; the altitude of one pyramid is $a\sqrt{2}$ and of the other $2a\sqrt{3}$.

*a* Find the ratio of their volumes. [4]

*b* Find the ratio of their lateral areas. [$8\frac{1}{2}$]

9 Find the lateral area and the volume of a frustum of a cone of revolution if the diameters of the bases are respectively 4″ and 10″ and the slant height is 5″. [4, $8\frac{1}{2}$]

10 The area of a zone whose altitude is 7 inches is 352 square inches; find in square inches the area of a spheric triangle on the same sphere as the zone if the angles of the triangle are 100°, 125° and 225°. [$12\frac{1}{2}$]

11 The dimensions of a rectangular parallelepiped are in the ratios 1:2:3 and a diagonal of the parallelepiped is $2\sqrt{14}$; find the total area and the volume of the solid. [$12\frac{1}{2}$]

12 State whether *each* of the following statements is true or false: [Write the letters *a*, *b*, *c*, *d*, *e* in a vertical column and then write the word *true* or *false* after each letter.] [$12\frac{1}{2}$]

*a* If each of two planes is perpendicular to a third plane, the first two planes are parallel.

*b* All the straight lines tangent to two spheres with equal radii form a cylindric surface.

*c* Four spheres may be so placed that each is tangent to each of the other three.

*d* The lateral surface of a cone of revolution may be spread out in the form of a sector of a circle.

*e* If each of three lines intersects the other two, then the three lines must lie in a plane.