SOLID GEOMETRY

Friday, January 21, 1927 — 9.15 a.m. to 12.15 p.m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Name the author of the textbook you have used in your study of solid geometry.

Answer eight questions, including not more than four from group I.

Group I

Do not answer more than four questions from this group.

1. Prove that two straight lines perpendicular to the same plane are parallel. \([12\frac{1}{2}]\)

2. Prove that the locus of points equally distant from the extremities of a straight line is the plane perpendicular to that line at its middle point. \([12\frac{1}{2}]\)

3. Prove that every section of a circular cone made by a plane parallel to its base is a circle the center of which is the intersection of the plane with the axis. \([12\frac{1}{2}]\)

4. Prove that the volume of a triangular pyramid is equal to one third the product of its base and its altitude. \([12\frac{1}{2}]\)

5. Prove that two symmetrical spheric triangles are equal. \([12\frac{1}{2}]\)

Group II

Irrational results may be left in the form of \(\pi\) and radicals unless otherwise stated.

6. What is the locus of a point equidistant from two intersecting planes and also at a given distance from the line of intersection of the planes? \([12\frac{1}{2}]\)

7. \(OABC\) is a triangular pyramid with vertex \(O\) and base \(ABC\). \(D\) is the mid-point of \(OB\) and \(E\) is the mid-point of \(OC\). Prove that the plane determined by \(A, D\) and \(E\) cuts the plane of the base \(ABC\) in a line parallel to \(BC\). \([12\frac{1}{2}]\)

8. The bases of two regular pyramids are equal squares with sides \(a\); the altitude of one pyramid is \(a\sqrt{2}\) and of the other \(2a\sqrt{3}\).

   a. Find the ratio of their volumes. \([4]\)

   b. Find the ratio of their lateral areas. \([8\frac{1}{2}]\)

9. Find the lateral area and the volume of a frustum of a cone of revolution if the diameters of the bases are respectively \(4\) \(\text{in}\) and \(10\) \(\text{in}\) and the slant height is \(5\) \(\text{in}\). \([4, 8\frac{1}{2}]\)

10. The area of a zone whose altitude is \(7\) \(\text{in}\) is \(352\) square inches; find in square inches the area of a spheric triangle on the same sphere as the zone if the angles of the triangle are \(100\) \(\text{deg}\), \(125\) \(\text{deg}\), and \(225\) \(\text{deg}\). \([12\frac{1}{2}]\)

11. The dimensions of a rectangular parallelepiped are in the ratios \(1:2:3\) and a diagonal of the parallelepiped is \(2\sqrt{14}\); find the total area and the volume of the solid. \([12\frac{1}{2}]\)

12. State whether each of the following statements is true or false: [Write the letters \(a, b, c, d, e\) in a vertical column and then write the word true or false after each letter.] \([12\frac{1}{2}]\)

   a. If each of two planes is perpendicular to a third plane, the first two planes are parallel.

   b. All the straight lines tangent to two spheres with equal radii form a cylindric surface.

   c. Four spheres may be so placed that each is tangent to each of the other three.

   d. The lateral surface of a cone of revolution may be spread out in the form of a sector of a circle.

   e. If each of three lines intersects the other two, then the three lines must lie in a plane.