

SOLID GEOMETRY

Monday, January 21, 1924—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry. The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Name the author of the textbook you have used in your study of solid geometry.

*Answer eight questions, including not more than four from group I.*

Group I

Do not answer more than four questions from this group.

- 1 Prove that if a line is perpendicular to each of two other lines at their point of intersection, it is perpendicular to the plane of these lines.  $[12\frac{1}{2}]$
- 2 Prove that any point in the plane bisecting the dihedral angle formed by two planes is the same distance from each plane.  $[12\frac{1}{2}]$
- 3 Prove that the volume of a triangular pyramid is equal to one third the product of its base and its altitude.  $[12\frac{1}{2}]$
- 4 Prove that every section of a circular cone made by a plane parallel to its base is a circle, the center of which is the intersection of the plane with the axis.  $[12\frac{1}{2}]$
- 5 Prove that the sum of the angles of a spheric triangle is greater than two, and less than six, right angles.  $[12\frac{1}{2}]$

Group II

*Irrational results may be left in the form of  $\pi$  and radicals unless otherwise stated.*

- 6 Find the locus of a point equidistant from two given points and at a given distance from a given plane.  $[12\frac{1}{2}]$
- 7 a If two of the angles of a spheric triangle on a sphere whose radius is 5 feet, are  $120^\circ$  and  $80^\circ$  respectively, find the value of the third angle if the area of the triangle is  $20\pi$  square feet.  $[8]$   
b On the same sphere, find the altitude of the zone having the same area as the triangle.  $[4\frac{1}{2}]$

- 8 The lateral area of a right circular cylinder whose radius is 4 feet is one half the total area. Find the lateral area of a similar cylinder that is eight times as large as the given cylinder.  $[12\frac{1}{2}]$
- 9 Prove that if a straight line is parallel to a plane, any plane perpendicular to the line is perpendicular to the plane.  $[12\frac{1}{2}]$
- 10 Find the lateral area and the volume of a frustum of a regular triangular pyramid, the sides of whose bases are 12 feet and 6 feet respectively and whose altitude is 5 feet.  $[12\frac{1}{2}]$
- 11 A right circular cone has its vertex in the surface of a sphere and its base is a section of the sphere made by a plane passing through the center.
  - a Find the ratio of the total surfaces of the two solids.  $[8]$
  - b Find the ratio of their volumes.  $[4\frac{1}{2}]$