**SOLID GEOMETRY**

**Monday, January 19, 1920—9.15 a.m. to 12.15 p.m., only**

Write at top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry. The minimum time requirement is two recitations a week for a school year or four recitations a week for half a school year.

Name the author of the textbook you have used in your study of solid geometry.

**Answer eight questions, including four from group I, two from group II and two from group III.**

**Group I**

**Answer four questions from this group.**

1. Prove that the intersections of two parallel planes with a third plane are parallel.

2. Prove that the sum of the face angles of a convex polyhedral angle is less than four right angles.

3. Prove that the volume of a triangular prism is equal to the product of its base and altitude.

4. Prove that the line connecting the center of a sphere and the center of a small circle of the sphere is perpendicular to the plane of the circle.

5. Prove that the surface of a sphere is equal to the product of the diameter by the circumference of a great circle of the sphere.

**Group II**

**Answer two questions from this group.**

6. A right circular cylinder is circumscribed about a sphere. Show that (a) the surface of the sphere is equivalent to \( \frac{1}{3} \) of the total surface of the cylinder, (b) the volume of the sphere is \( \frac{1}{6} \) of the volume of the cylinder. [No authorities (reasons) are required in answering this question.]

7. In the pyramid \( A-BCD \), prove that the lines joining in order the mid points of \( BC, AC, AD \) and \( BD \) form a parallelogram.

**Group III**

Answer two questions from this group.

9. The total surface \( (T) \) of a regular tetrahedron is \( 100\sqrt{3} \) square units. Find the altitude \( (H) \) and the volume \( (V) \) of the solid.

10. The frustum of a regular pyramid has square bases 8\" and 4\" respectively on a side, and an altitude of 15\". Find the altitude of an equivalent pyramid whose base is a mid section of the frustum.

11. A lune whose angle is 40\° is equivalent to a zone on the same sphere. Find the ratio of the altitude of the zone to the radius of the sphere.

12. Half of a regular hexagon inscribed (as shown in the drawing) in a semicircle whose radius is 12, is revolved about the diameter of the semicircle as an axis. Find the surface and volume generated by the semipolygon.