SOLID GEOMETRY
Friday, January 27, 1905 — 1.15 to 4.15 p. m., only

Answer eight questions but no more. If more than eight are answered only the first eight answers will be considered. Draw carefully and neatly each figure in construction or proof, using letters instead of numerals. Arrange work logically. Each complete answer will receive 12½ credits. Papers entitled to 75 or more credits will be accepted.

First division

1 Prove that if two straight lines are intersected by three parallel planes, their corresponding segments are proportional.

2 Prove that if two intersecting planes are each perpendicular to a third plane, their intersection is also perpendicular to that plane.

3 Prove that the sum of the face angles of any convex polyhedral angle is less than four right angles.

4 Give the formula for the volume of (a) a truncated triangular prism, (b) a spherical pyramid. Derive one of these formulas.

5 Complete and demonstrate the following: the area of the convex surface of a frustum of a cone of revolution is equal to . . .

6 Prove that in two polar triangles each angle of the one is the supplement of the opposite side of the other.

Note—Use π instead of its approximate value 3.1416.

Second division

7 Find the locus of a point equidistant from the three faces of a trihedral angle. Give proof.

8 Find the area and the volume of a solid generated by the revolution of an equilateral triangle about one of its sides as an axis, each side of the triangle being 4 inches.

9 A rectangle 6 feet × 4 feet is revolved about each of two of its adjacent sides; find the volume and the total surface of each solid generated.

10 The total surface of a right cylinder is S and the altitude is equal to the diameter of the base; find, in terms of S, the volume of the cylinder.

11 A regular hexagonal pyramid whose altitude is 8 inches and whose base is $54\sqrt{3}$ square inches, is cut by a plane parallel to the base; the area of this section is $\frac{1}{3}$ the area of the base. Find (a) the distance of this section from the base, (b) a side of this section.

12 Prove that sections of a sphere made by planes equally distant from the center of the sphere, are equal circles.