

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

**COURSE III**

Tuesday, June 22, 1993 — 1:15 to 4:15 p.m., only

Notice . . .

Calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

The “Reference Tables for Mathematics” and a formula sheet which you may need to answer some questions in this examination are stapled in the center of this booklet. Open the booklet and carefully remove the reference tables.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN**

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of  $\pi$  or in radical form. [60]

1 Express  $180^\circ$  in radian measure.

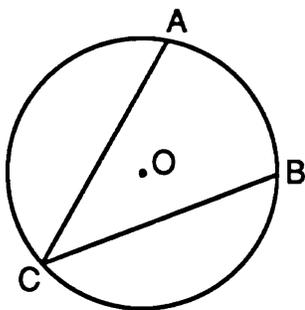
2 Solve for  $x$ :  $5^{3x-2} = 25$

3 In which quadrant do both the cosecant and secant functions have negative values?

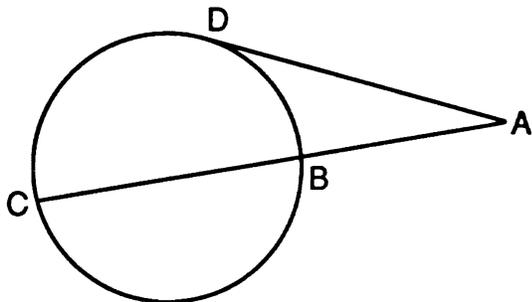
4 Express in simplest form in terms of  $i$ :  $\sqrt{-125}$

5 Solve for  $x$ :  $\sqrt{x+3} - 4 = 0$

6 In the accompanying diagram of circle  $O$ , the measure of  $\widehat{AB}$  equals  $80^\circ$ . What is the number of degrees in the measure of inscribed angle  $ACB$ ?



7 In the accompanying diagram,  $\overline{AD}$  is tangent to the circle at  $D$  and  $\overline{ABC}$  is a secant. Find  $m\angle A$  if  $m\widehat{DC} = 120$  and  $m\widehat{CB} = 170$ .



8 If  $\cos \theta = -\frac{1}{2}$  and  $\theta$  is not a third-quadrant angle, what is  $\sin \theta$ ?

9 Evaluate:  $\sin 270^\circ + \cos 60^\circ$

10 Express  $(1 + \sin \theta)(1 - \sin \theta)$  in terms of  $\cos \theta$ .

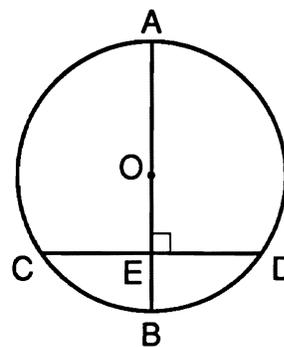
11 In  $\triangle ABC$ ,  $m\angle A = 30$ ,  $\sin B = \frac{3}{4}$ , and  $a = 8$ . Find the value of  $b$ .

12 Simplify:  $\frac{\frac{3}{x} - \frac{x}{3}}{\frac{1}{3} + \frac{1}{x}}$

13 In a circle whose radius is 4 centimeters, what is the length, in centimeters, of an arc intercepted by a central angle of  $2\frac{1}{2}$  radians?

Directions (14–35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

14 In the accompanying diagram of circle  $O$ , diameter  $\overline{AB}$  is perpendicular to chord  $\overline{CD}$  at  $E$ .



Which of these three statements is true?

- I.  $\overline{CE} \cong \overline{ED}$
- II.  $\widehat{CB} \cong \widehat{BD}$
- III.  $\widehat{AC} \cong \widehat{AD}$

- (1) I, only
- (2) I and II, only
- (3) I and III, only
- (4) I, II, and III

- 15 The graph of which equation has the same amplitude as the graph of the equation  $y = 2 \cos x$ ?
- (1)  $y = \sin 2x$                       (3)  $y = 2 \tan x$   
(2)  $y = \frac{1}{2} \cos 2x$                   (4)  $y = 2 \sin x$
- 16 If  $\angle A$  and  $\angle B$  are acute angles,  $\sin A = \frac{4}{5}$ , and  $\cos B = \frac{5}{13}$ , what is the value of  $\sin(A + B)$ ?
- (1)  $\frac{16}{65}$                                   (3)  $\frac{56}{65}$   
(2)  $\frac{46}{65}$                                   (4)  $\frac{63}{65}$
- 17 The value of  $\tan\left(\text{Arc sin } \frac{\sqrt{3}}{2}\right)$  is
- (1) 1                                        (3)  $\sqrt{3}$   
(2)  $\frac{\sqrt{2}}{2}$                                   (4)  $\frac{\sqrt{3}}{2}$
- 18 What is the value of  $\sum_{k=1}^4 (2k + 1)$ ?
- (1) 24                                        (3) 10  
(2) 15                                        (4) 4
- 19 Which transformation does *not* preserve orientation?
- (1)  $T_{5,-3}$                                 (3)  $D_3$   
(2)  $r_{y=x}$                                 (4)  $R_{0,90^\circ}$
- 20 What is the solution set of  $|2x - 3| \leq 1$ ?
- (1)  $x \leq 1$                                 (3)  $1 \leq x \leq 2$   
(2)  $x \geq 2$                                 (4)  $x \leq 1$  or  $x \geq 2$
- 21 If  $g(x) = \tan\left(x - \frac{\pi}{2}\right)$ , the value of  $g(\pi)$  is
- (1) 1                                        (3)  $\sqrt{3}$   
(2)  $\frac{1}{3}\sqrt{3}$                                 (4) undefined
- 22 The graph of the equation  $y = \left(\frac{1}{4}\right)^x$  lies in  
Quadrants
- (1) I and IV                                (3) III and IV  
(2) I and II                                (4) II and III
- 23 If  $x = 4ab^2$ , which expression is equivalent to  $\log x$ ?
- (1)  $\log 8 + \log a + \log b$   
(2)  $2(\log 4 + \log a + \log b)$   
(3)  $\log 8ab$   
(4)  $\log 4 + \log a + 2 \log b$
- 24 The domain for  $f(x) = x^2 - 3$  is  $0 \leq x < 4$ . The smallest value in the range of  $f(x)$  is
- (1) 0                                        (3) -3  
(2) 16                                        (4) 4
- 25  $\tan(-100^\circ)$  is equivalent to
- (1)  $\tan 80^\circ$                                 (3)  $-\tan 80^\circ$   
(2)  $\tan 10^\circ$                                 (4)  $-\tan 10^\circ$
- 26 The roots of the equation  $3x^2 + 2x - 1 = 0$  are
- (1) rational                                (3) imaginary  
(2) irrational                                (4) equal
- 27 What is the value of  $(x + 1)^{-\frac{2}{3}}$  if  $x = 7$ ?
- (1)  $-\frac{1}{4}$                                         (3) -4  
(2)  $\frac{1}{4}$                                         (4) 4
- 28 The coordinates of point A are (3,-1). What are the coordinates of A under the transformation  $(T_{2,5} \circ r_{x\text{-axis}})(A)$ ?
- (1) (-1,4)                                (3) (5,4)  
(2) (5,6)                                        (4) (-5,-4)
- 29 In a normal distribution, what percent of information falls within one standard deviation of the mean?
- (1) 34                                        (3) 68  
(2) 50                                        (4) 95
- 30 Which equation is equivalent to  $y = 3^x$ ?
- (1)  $\log 3 = x$                                 (3)  $\log_3 x = y$   
(2)  $\log_y x = 3$                                 (4)  $\log_3 y = x$
- 31 If  $\cos x^\circ = \sin(2x - 30)^\circ$ , a value of  $x$  can be
- (1) 20                                        (3) 50  
(2) 40                                        (4) 60

32 If a fair coin is tossed four times, the probability of tossing *exactly* three heads is

(1)  $\frac{1}{16}$

(3)  $\frac{1}{4}$

(2)  $\frac{1}{8}$

(4)  $\frac{1}{2}$

33 Which is the third term in the expansion of  $(2x - 3)^5$ ?

(1)  $180x^3$

(3)  $-270x^3$

(2)  $720x^3$

(4)  $-1080x^2$

34 In  $\triangle ABC$ ,  $a = 1$ ,  $b = 1$ , and  $c = \sqrt{2}$ . What is the value of  $\cos C$ ?

(1) 1

(3)  $\frac{1}{2}\sqrt{2}$

(2)  $\sqrt{2}$

(4) 0

35 The graph of the equation  $xy = 2$  is

(1) a hyperbola

(3) an ellipse

(2) a line

(4) a parabola

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 a On the same set of axes, sketch and label the graphs of the equations  $y = \sin \frac{1}{2}x$  and  $y = \frac{1}{2} \cos x$  for the values of  $x$  in the interval  $-\pi \leq x \leq \pi$ . [4.4]

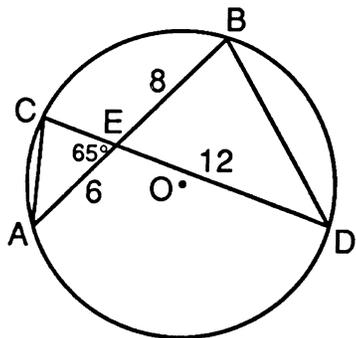
b In which interval is  $\sin \frac{1}{2}x$  always greater than  $\frac{1}{2} \cos x$ ? [2]

(1)  $-\pi \leq x \leq \frac{\pi}{2}$                       (3)  $0 \leq x \leq \frac{\pi}{2}$

(2)  $-\frac{\pi}{2} \leq x \leq 0$                       (4)  $\frac{\pi}{2} \leq x \leq \pi$

37 Find, to the nearest degree, all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$  that satisfy the equation  $2 \tan^2 x - 5 \tan x - 1 = 0$ . [10]

38 In the accompanying diagram of circle  $O$ , chords  $\overline{AB}$  and  $\overline{CD}$  intersect at  $E$ ,  $m\angle AEC = 65^\circ$ ,  $AE = 6$ ,  $EB = 8$ , and  $ED = 12$ .



Find:

a  $CE$  [2]

b  $BD$  to the nearest tenth [6]

c the area of  $\triangle EBD$  to the nearest tenth [2]

39 The table shows the average snowfall in centimeters recorded one winter at a ski resort over a period of days.

Snowfall (in cm)	Frequency
18	6
19	4
20	4
21	3
24	5
26	3

a Using this set of data, find the standard deviation of the snowfall to the nearest tenth. [6]

b The probability that it snows on a given day during the winter at the resort is  $\frac{2}{3}$ . Find the probability that it will snow at least three days during a five-day winter stay at the resort.

[4]

40 a Sketch and label the graph of the equation  $y = 3^x$ . [3]

b On the same set of axes, sketch and label the reflection of the graph of  $y = 3^x$  in the line  $y = x$ . [3]

c Write the equation for the reflected graph sketched in part b. [2]

d Using the graph sketched in part b, describe the behavior of the graph in Quadrant IV as  $x$  approaches 0. [2]

GO RIGHT ON TO THE NEXT PAGE.

41 a For all values for which the fraction is defined, simplify:

$$\frac{x^{-1} - y^{-1}}{y^{-2} - x^{-2}} \quad [5]$$

b Solve for x:

$$\frac{2x^2}{x^2 - 1} - \frac{3}{x + 1} = \frac{x}{x - 1} \quad [5]$$

42 a For all values of  $\theta$  for which the expressions are defined, prove the following is an identity:

$$\frac{1 + \cos 2\theta}{\sin 2\theta} = \cot \theta \quad [5]$$

b Using logarithms, find  $\frac{(\sqrt[3]{100})}{2}$  to the nearest tenth. [5]