## HIGH SCHOOL MATHEMATICS: COURSE III-JUNE 1983 (1)

## Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers on a separate sheet. Where applicable, answers may be left in terms of  $\pi$  or in radical form.

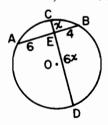
- 1 Express the sum of  $\sqrt{-49}$  and  $2\sqrt{-16}$  as a monomial in terms of *i*.
- 2 Find the value of  $\sum_{n=1}^{3}$  (2n-1).
- 3 Express  $\frac{1}{3 \sqrt{3}}$  as an equivalent fraction

with a rational denominator.

4 Perform the indicated operations and express in simplest form:

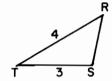
$$\left(\frac{x+y}{x}\right)\left(\frac{xy}{x^2+2xy+y^2}\right)$$

- 5 Solve for  $y: \frac{3}{y} = 2 + \frac{5}{y}$
- 6 In the diagram below, chords  $\overline{AB}$  and  $\overline{CD}$  of circle O intersect in E. If AE = 6, EB = 4, CE = x, and ED = 6x, find CE.



7 Given the set of functions: {sin x,cos x,tan x}. What is the probability that a function chosen at random from this set is positive in both Quadrants I and III?

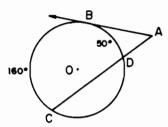
- 8 If  $\log n = 0.5459$ , find n to the nearest thousandth.
- 9 Find the numerical value of  $\sin (-270^{\circ})$ .
- 10 If tan  $A = \frac{1}{3}$ , find the value of tan 2A.
- 11 A diagonal is drawn in a rectangle whose dimensions are 10 cm by 50 cm. Find the tangent of the angle formed by the diagonal and the longer side.
- 12 Express the product 5i(3i 2) as a complex number in a + bi form.
- 13 In the accompanying diagram of  $\triangle RST$ , ST = 3 and RT = 4. If  $m \angle T = 30$ , find the area of  $\triangle RST$ .



- 14 If  $\cos x = \frac{1}{8}$  where x is an acute angle, find the value of  $\cos \frac{x}{2}$ .
- 15 Solve the equation  $\sqrt{2 \sin x + 7} = 3$  for the smallest positive value of x.
- 16 If  $3^{x+2} = 9^x$ , what is the value of x?
- 17 If  $f(x) = 3 \cos \frac{x}{3}$ , find  $f(\pi)$ .
- 18 In  $\triangle ABC$ , a = 4, b = 5, and  $\cos C = \frac{1}{8}$ . Find the length of side c.

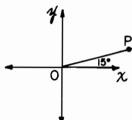
19 In the accompanying diagram,  $\overrightarrow{AB}$  is tangent to circle O at B and  $\overrightarrow{ADC}$  is a secant.

If  $\widehat{mBD} = 50$  and  $\widehat{mBC} = 160$ , find  $m \angle A$ .



Directions (20-35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

- 20 In the accompanying diagram,  $\overline{OP}$  forms an angle of 15° with the x-axis. The reflection of  $\overline{OP}$  in the y-axis is  $\overline{OP}'$ . What is the measure of  $\angle POP'$ ?
  - (1) 30
  - (2) 75
  - (3) 135
  - (4) 150



- 21 A set of measures that follows a bell curve has a mean of 50 and standard deviation of 5. Approximately what percent of the measures fall between 45 and 55?
  - (1) 34

(3) 95

(2) **68** 

- (4) 98
- 22 The expression  $\sin \theta(\csc \theta \sin \theta)$  is equivalent to
  - (1) 1

(3)  $\tan \theta - 1$ 

 $(2) \cos \theta$ 

(4)  $\cos^2 \theta$ 

(1) 
$$\{(x,y)|y = \sin x\}$$
 (3)  $\{(x,y)|y = 4\}$ 

$$(2) \{(x,y)|y=-x\} \qquad (4) \{(x,y)|x=4\}$$

24 The graph of the equation  $y = x^2 + 2x - 8$  intersects the x-axis at

$$(2)$$
 -2 and 4  $(4)$  2 and 4

25 The expression  $\log 3x$  is equivalent to

(1) 
$$(\log 3)(\log x)$$
 (3)  $\log 3 + \log x$   
(2)  $3 \log x$  (4)  $\log (3 + x)$ 

26 A fair die is tossed three times. The probability of obtaining exactly 2 fives is

(1) 
$$\frac{1}{72}$$
 (3)  $\frac{25}{72}$ 

$$(2) \ \frac{5}{72} \qquad \qquad (4) \ \frac{67}{72}$$

27 The roots of the equation  $-x^2 + x + 6 = 0$  are

- (1) real, rational, and unequal
- (2) real, irrational, and unequal
- (3) real, rational, and equal
- (4) imaginary

28 If  $y = \cos(\operatorname{Arc sin} \frac{\sqrt{3}}{2})$ , then y is equal to

(1) 
$$\frac{\sqrt{3}}{2}$$
 (3) 30°

(2) 
$$\frac{1}{2}$$
 (4)  $60^{\circ}$ 

29 A circle has a radius of 6 centimeters. What is the number of radians in a central angle which has an arc of length 12 centimeters?

$$(1) \frac{1}{2}$$
 (3) 18

30 What is the solution set of the equation

$$|3x + 2| = 5$$
?

 $(1) \{1\}$ 

(3)  $\{1, -\frac{7}{2}\}$ 

 $(2) \left\{ \frac{7}{2} \right\}$ 

- $(4) \{-1,\frac{7}{2}\}$
- 31 Which is the fourth term in the expansion of  $(x + 3)^5$ ?
  - (1)  $270x^2$

(3) + 405x

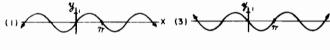
(2)  $135x^2$ 

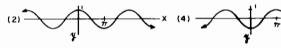
- (4) + 135x
- 32 The value of  $\frac{1}{9^{-3}}$  is
  - $(1) \frac{1}{9}$

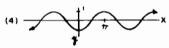
(3) 6

 $(2) \frac{1}{6}$ 

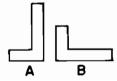
- (4) 8
- 33 Which graph represents the reflection over the x-axis of the curve  $y = \sin x$ ?







- 34 As angle  $\theta$  increases from 0 to  $\pi$ , the value of  $\cos \theta$  will
  - (1) decrease, only
  - (2) increase, only
  - (3) decrease then increase
  - (4) increase then decrease
- 35 Figure B is the image of figure A under which single transformation?
  - (1) line reflection
  - (2) translation
  - (3) rotation
  - (4) glide-reflection



## Part II

Answer four questions from this part. Show all work unless otherwise directed.

- 36 a On the same set of axes, sketch and label the graphs of  $y = \sin x$  and  $y = 2 \cos x$  in the interval  $-\pi \le x \le \pi$ . [8]
  - b For how many values in the interval  $-\pi \le x \le \pi$  does sin  $x = 2 \cos x$ ? [2]
- 37 A class of students obtained the following results on a test:
  - 4 students received 90%
  - 5 students received 80%
  - 8 students received 70%
  - 3 students received 60%

For these scores, find the:

- a mean [3]
- b standard deviation to the nearest tenth [7]
- 38 a Solve the equation  $x^2 + 2x + 10 = 0$  and express the roots in a + bi form. [4]
  - b Prove the identity:

$$\sin 2x = \frac{2 \tan x}{1 + \tan^2 x} \quad [6]$$

- 39 a Given the function  $f = \{(x,y)|y = \log_2 x\}$ .
  - (1) Sketch the graph of f. [4]
  - (2) Write an equation for f<sup>-1</sup>, the inverse of f. [2]
  - b Using logarithms, find  $\sqrt[3]{432}$  to the nearest hundredth. [4]
- 40 Given points A(0,0), B(8,6), and C(8,0).
  - a Graph  $\triangle ABC$ . [1]
  - b Graph and state the coordinates of  $\triangle A'B'C'$ , the image of  $\triangle ABC$  after the transformation  $D:(x,y) \to (\frac{3}{2}x, \frac{3}{2}y)$ . [3]

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- c Graph and state the coordinates of  $\triangle A''B''C''$ , the image of  $\triangle ABC$  after a reflection in the line x = 2. [3]
- d Graph and state the coordinates of  $\triangle A'''B'''C'''$ , the image of  $\triangle ABC$  under the transformation  $G:(x,y) \rightarrow (x+3,y-7)$ . [3]
- 41 In  $\triangle ABC$ ,  $m \angle B = 38^{\circ}$ ,  $m \angle C = 56^{\circ} 20'$ , and a = 12. Find the length of side c to the nearest integer. [10]
- 42 In the accompanying diagram, ABCD is inscribed in circle O,  $m\angle COD = 70$ ,  $\overrightarrow{AOD}$  is a diameter,  $\overrightarrow{PB}$  is tangent to circle O at B,  $\overrightarrow{PCOE}$  is a secant, and  $\overrightarrow{mBA} = \overrightarrow{mCD}$ .

Find:  $a \quad \widehat{mBC} \quad [2]$   $b \quad \widehat{m \angle PBC} \quad [2]$   $c \quad \widehat{m \angle A} \quad [2]$   $d \quad \widehat{m \angle BPE} \quad [2]$  $e \quad \widehat{m \angle AFE} \quad [2]$ 

